# A note on the impact of Basel II on banking and economic crises

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**Abstract**

In this paper we analyze the impact of the risk sensitivity of capital requirements in Basel II *during a crisis,* both in the real economy and in the banking activity. We argue that Basel II will reduce the probability and severity of banking crises, but it might increase the negative impact of an economic downturn into the real economy.

We present a continuous time model to study the effects of a risk sensitive capital requirements rule, such as Basel II, on banks' risk taking behaviour. The results show that for low asset values banks switch from high to low risk levels in order to reduce the regulatory risk weights of their assets and, as a consequence, boost their capital ratios. The more risk sensitive the capital requirements rule, the sooner the switch from high risk to low risk assets. We hypothesize that such low risk assets are also characterized by (i) a higher liquidity and (ii) a lower proportion of investment in credit to the industry.

After reviewing the literature on the impact of credit crunches and liquidity in both the real economy and the banking sector, we argue that the shift to low risk assets during crises implied by the risk sensitive capital requirements rule of Basel II will increase the liquidity of banks' assets, but will reduce the availability of credit to the industry. In an economic downturn the higher liquidity will reduce the probability of a banking crisis through a contagion mechanism. However, the decline in credit availability to firms and businesses, particularly small firms and those with high associated risk weights, will further weaken economic activity through a so-called credit crunch.

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# 1 Model

In what follows we use a continuous time model where banks' asset values follow a geometric Brownian motion, banks can costlessly and dynamically change their risk levels and they are subject to a capital requirements rule. In order to verify banks satisfy such a rule, the unobservability of capital and risk levels by the supervisor requires it has to audit the bank at certain, stochastic, dates.

The model belongs to a line of research started by Merton (1978), applying continuous time models for banking regulation and supervision. Merton's model is the standard in the literature because of its simplifying assumptions,[1] which subsequent papers have relaxed. See, among others, Milne and Whalley (2001), Bhattacharya et al. (2002), Peura (2003), Dangl and Lehar (2004), Decamps, Rochet and Roger (2004), Elizalde (2005) and Keppo and Peura (2005).[2]

Consider a bank which, at each time $t$ in which it is open, has an asset size equal to $A_t$, funded by deposits, $D$ and capital, $A_t - D$. The bank is owned by risk neutral equityholders who enjoy limited liability and who provide the bank's capital at a cost $\nu$.

The time-horizon of the bank's equityholders, which we also refer to as maturity, is considered to be finite and denoted by $T$. If still open at $T$, the bank is liquidated and equityholders receive the bank's asset value $A_T$ minus the face value of deposits $D$ if that difference is positive and, due to limited liability, zero otherwise. The bank's equityholders take their decisions in order to maximize the expected stream of cash flows derived from equity up to time $T$.

In order to simplify the presentation, we assume that there are no intermediation costs, that deposits are fully insured and remain constant over time, and that the deposit insurer does not charge banks any deposit insurance premium.

[1] Among others: banks can not choose their risk levels, the dividend/recapitalization policy is exogenous and costless, the audit frequency of the supervisor is constant and independent of the banks' rating levels, ...

[2] Elizalde (2005) reviews this literature and the contributions of each of the cited works.

The bank's asset value $A_t$ follows a geometric Brownian motion:

$$\frac{dA_t}{A_t} = (\mu - \delta)\, dt + \sigma_t dW_t, \tag{1}$$

where $\mu$ is the total expected return, $\delta$ is the payout ratio (fraction of the asset paid out to security holders), $\sigma_t$ is the asset's relative instantaneous volatility and $W_t$ is a standard Brownian motion.

The expected return $\mu$ and payout ratio $\delta$ are constant across time. At any time $t$, shareholders can choose between two risk levels $\sigma_t$ : high $\sigma_H$ and low $\sigma_L$, where $\sigma_L < \sigma_H$. We follow Ross (1997) and Leland (1998) and assume risk switching is costless.[3]

The choice of the risk level does not modify the instantaneous asset return. Although in practice one might associate a higher rate of return with a higher risk level, we abstract from that in order for the bank's risk taking decisions to be based only on risk considerations.

At time $t$, the bank has available an amount $\delta A_t$ to share among equityholders and depositors. Depositors receive a continuous deposit rate $r_d$ as long as the bank is open. The rest, $\delta A_t - r_d D$, is kept by equityholders as dividend payment. $\delta A_t - r_d D$ is negative for values of $A_t$ lower than $\frac{r_d D}{\delta}$. In those cases, equityholders can either inject money to keep the bank open or, voluntarily, close the bank and stop paying depositors.

Given their asset $A_t$ and risk $\sigma_t$ level, banks are required to hold a minimum capital level according with a capital requirements rule specified below. While the bank's asset $A_t$ and risk $\sigma_t$ levels are observed continuously by equityholders, the supervisor can only observe them auditing the bank. Audit times are stochastic and follow a Poisson distribution characterized by an intensity parameter $\lambda$. If the supervisor discovers, in one of the audits, the bank not complying with the capital

[3]Dangl and Lehar (2004) assume costly risk switching. As shown by the mentioned papers, if banks are given the option to choose a risk level in a given interval, the optimal strategy is to take either the lowest or the highest possible risk level. That is the reason we consider the choice between two risk levels.

requirements rule, it takes control of the bank. Otherwise, no action is taken. Form the viewpoint of the equityholders, intervention is equivalent to closure.

The capital requirements rule that banks must satisfy (both under Basel I and Basel II) is

$$\frac{\text{Capital}}{\text{Risk Weighted Assets}} \geq \rho, \tag{2}$$

where $\rho$ is fixed to 8% in both Basel accords, but which national regulators can vary.

The only part of the previous rule which changes from Basel I to Basel II is the denominator, which is called Risk Weighted Assets ($RWA$) and is computed as the volume of assets in the bank portfolio weighted by their risk level.

We distinguish between risk sensitive and risk insensitive capital requirements rule. In a risk insensitive one, RWA are similar for the two levels of risk, $\sigma_L$ and $\sigma_H$. A risk sensitive capital requirements rule increases (decreases) the risk weights of high (low) risk assets.

We consider the following specification for $RWA$

$$RWA\left(\sigma_t\right) = \begin{cases} \left(1+\theta\right) A_t & \text{if } \sigma_t = \sigma_H, \\ \left(1-\theta\right) A_t & \text{if } \sigma_t = \sigma_L, \end{cases} \tag{3}$$

where $\theta > 0$ is the risk sensitivity parameter: the higher $\theta$ the more risk sensitive the capital requirements rule.[4] If $\theta = 0$ the capital requirements rule is risk insensitive; in which case the asset risk weight is 100% independently of the risk level. In Basel I a 100% risk weight is given to, for example, corporate lending.

The minimum capital requirement can be mapped into a minimum asset level $A_C$ such that if $A_t < A_C$ the bank does not satisfy the capital requirements rule. $A_C$ is a function of the risk level $\sigma_t$ and of the risk sensitivity parameter $\theta$ :

$$A_C\left(\sigma_t, \theta\right) \begin{cases} \frac{D}{1-\rho(1-\theta)} & \text{if } \sigma_t = \sigma_H, \\ \frac{D}{1-\rho(1+\theta)} & \text{if } \sigma_t = \sigma_L, \end{cases} \tag{4}$$

in such a way that

[4] Any other specification for $RWA(\sigma_t)$ won't change the results as long as $\frac{\partial RWA}{\partial \sigma_t} > 0$.

- In a risk insensitive capital requirements rule the supervisor's closure level $A_C$ does not depend on the bank's risk $\sigma_t$ :

$$A_C(\sigma_L, \theta = 0) = A_C(\sigma_H, \theta = 0). \tag{5}$$

- In a risk insensitive capital requirements rule

$$\frac{\partial (A_C(\sigma_H, \theta > 0) - A_C(\sigma_L, \theta > 0))}{\partial \theta} > 0. \tag{6}$$

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$$A_C(\sigma_L, \theta > 0) < A_C(\sigma_L, \theta = 0) = A_C(\sigma_H, \theta = 0) < A_C(\sigma_H, \theta > 0). \tag{7}$$

Although Basel I already incorporates some degree of risk sensitivity, the main contribution of Basel II (Pillar 1) is an increase of the risk sensitivity of the capital requirements rule. "*All of us have strongly wished for greater risk sensitivity.* The lack of differentiation of risk in the original Capital Accord was heavily criticized by banks and observers."[5]

# 2 Numerical results

Equity is an "exotic" option on the bank's assets. Exotic because it differs from a plain vanilla call option in the following: (i) it generates a continuous cash flow $\delta A_t - r_d D$ (negative or positive); (ii) it allows equityholders to continuously change the volatility of the diffusion process followed by the asset level; (iii) it allows equityholders to exercise the option at any time before maturity with a zero payoff; and (iv) the option is exogenously exercised by the supervisor if the bank is discovered not satisfying the capital requirements rule at an audit time. In that case, the holders receive a zero payoff.

At each time $t$ and given the bank's asset level $A_t$, the bank's equityholders decide the risk level $\sigma_t$ and whether or not to close the bank. Since once closed, the bank

[5] Crockett (2002), former General Manager of the Bank of International Settlements. Emphasis added.

cannot be re-opened, there exists an asset level $A_{E,t}$ such that the bank's equityholders will close the bank if $A_t \leq A_{E,t}$.

The numerical solution method used to solve for the value of equity and the asset levels at which the bank's equityholders close the bank and switch between risk levels is outlined in Appendix A. It consists on an explicit finite differences method, which is a standard procedure when valuing American options and consists on discretizing the time to maturity and the asset value into a two-dimensional grid. Starting from the boundary condition at maturity $T$, which guarantees that the value of equity is $\max\{A_T - D, 0\}$, we work backwards solving for the value of equity at each point in the grid.

Using the parameters in Appendix B and a risk insensitive capital requirements rule ($\theta = 0$), the resulting equityholders' decisions can be summarized as follows: (i) the bank is closed if $A_t \leq A_{E,t}\,(< A_C)$, and (ii) the optimal risk strategy is given by

$$\sigma_t^* = \begin{cases} \sigma_L & \text{if } A_t \leq A_{S,t}, \\ \sigma_H & \text{if } A_t > A_{S,t}. \end{cases} \tag{8}$$

There exists a unique risk switching level $A_{S,t}\,(> A_C)$ such that the bank prefers high risk below it and low risk above it.

In a risk sensitive capital requirements rule, $\theta > 0$, the risk switching level $A_{S,t}$, such that banks take low risks if the asset level is above it, remains; although it is reduced with respect to the case with $\theta = 0$, $A_{S,t}\,(\theta > 0) < A_{S,t}\,(\theta = 0)$. The following Table shows the impact of the model's parameters on the risk switching level $A_{S,t}$, which holds for any risk sensitivity parameter $\theta$. A negative sign represents that banks switch to low risk for a lower asset value, indicating a more prudent risk strategy.

| Parameter | $\frac{\partial A_{S,t}}{\partial(\text{Parameter})}$ |
|---|---|
| $\delta$ | $+$ |
| $r_d$ | $-$ |
| $\mu$ | $-$ |
| $\nu$ | $+$ |
| $\lambda$ | $-$ |
| $T-t$ | $+$[6] |
| $\sigma_L$ | $-$ |
| $\sigma_H$ | $+$ |

Unlike in the risk insensitive capital requirements rule case in which banks always prefer high risk for all asset levels below $A_{S,t}$, when the capital rule is risk sensitive banks prefer to take high risks for all asset levels below $A_{S,t}$ except for an interval of asset values between the supervisor closure levels, $A_C(\sigma_L, \theta > 0)$ and $A_C(\sigma_H, \theta > 0)$. If the audit frequency is sufficiently high, high risk is preferred for all asset levels in the interval $[A_C(\sigma_L, \theta > 0), A_C(\sigma_H, \theta > 0)]$, and therefore the optimal risk strategy is given by

$$\sigma_t^* = \begin{cases} \sigma_H & \text{if } A_t \in [A_{E,t}, A_C(\sigma_L, \theta > 0)], \\ \sigma_L & \text{if } A_t \in [A_C(\sigma_L, \theta > 0), A_C(\sigma_H, \theta > 0)], \\ \sigma_H & \text{if } A_t \in [A_C(\sigma_H, \theta > 0), A_{S,t}], \\ \sigma_L & \text{if } A_t > A_{S,t}. \end{cases} \tag{9}$$

Banks switch back to high risk when the asset value falls below $A_C(\sigma_L, \theta > 0)$ because for those asset values for which neither with high nor with low risk levels they satisfy the capital requirements rule. There is nothing to lose and they "gamble for resurrection".

Figure (1) represents the bank's risk taking decisions for a risk sensitive and a risk insensitive capital requirements rule.

[6] As in Ross (1997), the higher the time to maturity $T-t$ the more willing are the bank's equityholders to take high risks.

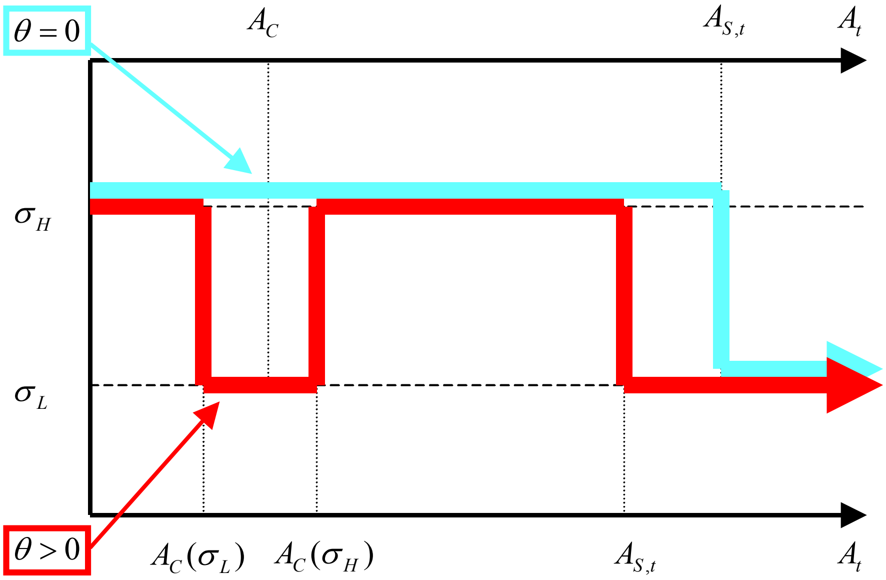


Figure 1: Bank's risk taking decisions for a risk sensitive ($\theta > 0$) and a risk insensitive ($\theta = 0$) capital requirements rule as a function of the asset value.

# 3 Discussion

## 3.1 Credit crunch

Although somehow speculatively, this Section relates our results with the so called "credit crunch" literature, arguing that the more risk sensitive the capital requirements rule the higher the probability of a credit crunch. Let us first briefly review that literature.

When banks face a period of financial distress, for example triggered by a general macroeconomic downturn, and their capitalization ratios reduce and dangerously approach the regulatory minimum capital requirements, they have two alternatives in order to boost their capital ratios. First, they can issue new equity and increase their capital levels, increasing the numerator of the capital rule (2). Second, they can reduce their assets risk levels in order to reduce their risk weights, reducing the denominator of the capital rule (2). Dahl and Shrieves (1990), Jacques and Nigro

(1997), and Aggarwal and Jacques (1998) present evidence that banks switch from high to low risk-weighted assets when their capital levels approach the regulatory ones.

The idea behind a credit crunch is that if banks fear their capital ratios will decrease to levels close or even lower than the regulatory ones, they will switch from high to low risk assets by reducing their credits and loans to the industry (which receive the highest risk weights). A reduction in the credit to the private sector can worsen economic conditions and make an economic crisis longer and deeper. The negative effect on business activity derived from a credit crunch will be particularly damaging on small businesses which rely heavily on banks for credit, in contrast with large firms, which have a higher access to direct market funds through equity or debt issuance.[7]

The Council of Economic Advisors (1992) defines a credit crunch as a “situation in which the supply of credit is restricted below the range usually identified with prevailing market interest rates and the profitability of investment projects”. Bernauer and Koubi (2004) argue that a credit crunch occurs “when banks refuse to make loans even though borrowers are willing to pay the stated interest rate or even a higher rate. Banks thus restrict the size of loans made to less than the full amount.”

Andrew Crockett, former General Manager of the Bank of International Settlements, acknowledged that an issue to be considered in reviewing capital adequacy regulation is the relationship between capital holding and the economic cycle:

> “When the macro-economic climate worsens, more loans go bad and more provisions have to be made. This naturally reduces capital ratios and induces banks to be cautious about granting new loans. A decline in credit availability, in turn, can further weaken economic activity and thus provide an additional twist in the vicious circle. Unless capital holding is well above the minimum in good times, capital adequacy rules will

[7] See Brewer et al. (1996).

constrain lending and may result in a credit crunch when a downturn occurs."[8]

Borio, Furfine and Lowe (2001) analyze the evolution of risk weighted assets during the 1990s in ten different countries[9] and conclude that "in the aftermath of the banking crises, risk-weighted assets fell more strongly than total assets, as banks shifted their portfolios away from commercial lending (which has a relatively high risk weight) towards residential mortgages and public sector securities (both of which have relatively low risk weights)."

To test the impact of a capital requirements rule on banks' risk shifting behavior during crisis times, several studies have focused on the first half of the 1990s, when both the introduction of Basel I and an economic slowdown simultaneously took place in many countries around the world. Basel I represented, in most cases, both an increase in regulatory capital $\rho$ as well as the first attempt to introduce risk sensitivity on the capital requirements rule.

Hancock and Wilcox (1998) and Chiuri, Ferri and Majnoni (2001) find evidence that the increase in capital requirements and the adoption of risk sensitive capital requirements rules reduced bank credit to high risk weight industries. However, Bernauer and Koubi (2004) find significant differences across countries. Goodhart, Hofmann and Segoviano (2004), after analyzing the US, Mexico and Norway cases, find that banks increased their capital levels by partially shifting to low risk-weighted assets, and, cautiously, conclude that this *may* have caused a reduction in the credit supply in the economy. They recognize the difficulty of proving that a shift to low risk-weighted assets by banks is the cause of a reduction in economic activity, "mainly due to the problem with identifying and separating credit-demand and credit-supply movements." Finally, Barajas, Chami and Cosimano (2005) find, in a sample of Latin American and Caribbean counties over the period 1987 to 2000, that "both bank capi-

[8] Crockett (1998).

[9] United States, Japan, Germany, Italy, United Kingdom, Spain, Australia, Sweden, Finland and Norway.

talization and lending activities in Latin America increased after Basel. Consequently, Basel did not seem to lead to an overall credit decline."

Basel Committee on Banking Supervision (1999) presents an extensive review of the previous literature, summarizing the findings as follows:

> "The overall message from the empirical literature and the data is that, at least initially, the introduction of formal minimum capital requirements across the G-10 appears to have induced relatively weakly capitalized institutions to maintain higher capital ratios. At the same time, however, there is some evidence that bank capital pressures during recent cyclical downturns in the U.S. and Japan may have limited bank lending in those periods and contributed to economic weakness in some macroeconomic sectors. All of these effects may well have reflected both regulatory and market pressure on banks to maintain ratios at least as high as the minimum."

Therefore, empirical evidence suggests that Basel I induced banks, when faced with an economic slowdown, to shift their portfolios from high to low risk-weighted assets, which probably caused a credit crunch exacerbating the negative effects of the crisis.

Besides regulatory reasons, banks may want to switch to low risk assets and boost their capital ratios to signal the market their soundness and financial health. In that respect, Borio, Furfine and Lowe (2001) argue that

> "The experience of the countries that had banking system problems in the early 1990s is illustrative here. Soon after the problems were recognized, the management and in some cases the new owners of the banks made a concerted effort to establish quite high capital ratios. This was not so much driven by the requirements of the supervisors, although in some countries did play a role, but rather by a belief that, after experiencing

problems, the banks needed to demonstrate their financial strength and their commitment to better risk management. One way of doing so was to report a high capital ratio, even if this meant severely cutting back the size of the balance sheet and sacrificing long-term banking relationships."

Since the model analyzed in this paper do not include any signaling or market effect derived from the level of banks' capital level, we are able to isolate the impact of capital regulation on risk shifting. Our numerical results show that the higher the risk sensitivity of the capital requirements rule, the higher is the range of asset values for which banks will switch to low risky assets when the asset value is low enough as to make the capital requirements binding were the bank to take high risk.

In Basel I, the risk weights for different assets range from 0% to assets such as cash and claims on central governments to 100% to assets such as claims on the private sector and real state investments. In Basel II (standardized approach) the same pattern of risk weighted assets holds, although with a higher risk sensitivity.[10] Low risk weighted assets correspond to assets which do not represent a direct investment in an economy's main productive and commercial activities, which involve loans to the industry and which, in turn, have a higher risk weight.

In light of the higher risk sensitivity of the new capital requirements rule in Basel II, the model suggests that although Basel II will reduce banks' risk levels in good times (by a reduction in $A_{S,t}$), it will make a credit crunch more probable than Basel I whenever the next crisis comes around, because it will make banks switch from high to low risk-weighted assets at an earlier time and for a wider range of asset values. Therefore, Basel II will reduce the probability of a banking crisis when we are not in one, but it might increase its negative effects on economic activity if we get into one.

[10] See Basel Committee on Banking Supervision (1988 and 2004).

## 3.2 Liquidity

It is a reasonable assumption to consider low risk assets to be more liquid than high risk assets. In fact, Basel I and Basel II (standardized approach) give a higher risk weight to assets which, not only are riskier, but which are clearly less liquid.

In that case, our numerical results would suggest that the higher the risk sensitivity of the capital requirements rule, the earlier banks will switch to low-risk highly-liquid assets when their capital ratios deteriorate, because $A_C(\sigma_H, \theta > 0)$ increases. According with the theoretical model presented by Cifuentes, Ferrucci and Shin (2005), the higher the banks' assets liquidity the lower the probability of a banking crisis.[11]

Goodhart (2004) argues that recent banking regulations have put most of their attention in capital requirements, forgetting about the role of liquidity to reduce banking crises and deal with them when they hit:

> "Banks' holdings of liquid assets not only protects other commercial banks, it also protects the monetary authorities, and helps them to maintain systemic stability. The more liquid assets a bank has, the longer it can sustain adverse clearings. That provides a breathing space, and in cases of financial crises, time is of the essence. Time is necessary to gather and transmit information, and to agree on the best course of procedure. It is liquid assets, not capital, that provides time in crises."

As long as low risk assets are more liquid, a risk sensitive capital rule addresses Goodhart's (2004) concerns: in a situation of decreasing banks' asset values, the more risk sensitive the capital rule, the earlier banks switch to low risk and therefore highly liquid assets. This will reduce the probability of a contagious banking crisis and will provide supervisors the "breathing space" needed to take the necessary measures to prevent the crisis.

[11] When banks are hit by a negative shock which reduces their asset values, they are forced to sell some of their assets to meet their contractual obligations. The higher the liquidity of those assets, the lower the subsequent reduction on the assets market price derived from those sales.

A word of caution over the previous argument is due in light of recent research suggesting that "an increase in asset liquidity in times of crisis, paradoxically, reduces stability" (cf. Wagner 2005), because banks react to higher liquidity increasing their risk levels.

# 4 Conclusion

We have analyzed the impact of the risk sensitivity of the capital requirements rule on banks' risk taking decisions. We show that in a risk insensitive capital rule there is a unique risk switching asset level above which banks prefer low risk, while preferring high risk otherwise. Such a risk switching level is generally well above asset levels at which banks do not satisfy the capital requirements.

A risk sensitive capital requirements rule reduces the risk switching level mentioned in the last paragraph, which we interpret as a reduction in banks' risk taking incentives for high asset values.

In addition, a risk sensitive capital requirements rule induces banks to switch from high to low risk levels when the asset value falls below the level at which the bank does not satisfy the minimum capital level implied by a high risk asset portfolio. We have suggested two possible consequences of such a risk switching. First, it may imply a switch from credit to commercial and industrial firms towards less risky asset holdings such as cash or government securities. Relating this fact with the credit crunch literature, we argued that although a more risk sensitive capital requirements rule reduces banks' risk taking incentives when asset levels are high, it increases the range of (low) asset values for which banks switch from high to low risk-weighted assets and, therefore, might increase the probability of a credit crunch, further weakening economic activity.

However, low risk assets can also be interpreted as more liquid than high risk assets. In that case the switch from high to low risk-weighted assets for (low) asset values will increase the liquidity of the banking system and therefore will reduce the probability of a negative shock spreading across banks.

The liquidity argument implies that the severity of an economic downturn for banks will be lower. Basel II will probably reduce the number of closures and the severity of a crisis for banks. However, the credit crunch argument implies that the crisis will have a stronger impact in the real economy, specially in commercial and industrial business with little access to capital markets.

# Appendix

## A Numerical solution for the value of equity

This Appendix presents the numerical solution method for the value of equity, the asset value at which equityholders decide to voluntarily close the bank, and risk switching asset levels. All the previous levels are a function of time to maturity $T-t$. If the bank is still open at $T$, equityholders receive the positive difference between the asset value $A_T$ and the face value of deposits $D$. We solve the model for a given time $t<T$, computing the value of equity for assets in the interval $A_t \in [A_1, A_N]$.

In order to make the exposition as clear as possible, we present the solution method in two stages. First we consider the case in which equityholders do not have the option neither to voluntarily close the bank nor to change the risk level. Second, we introduce the closure option and the risk switching option.

### A.1 No closure and risk switching options

We fix a risk level $\sigma$ which implies a supervisor closure level $A_C$. Let $F(A,t)$ denote the value of equity for an asset value $A$ at time $t$. For simplicity, we have dropped the subindex $t$ from the asset value $A_t$. It can be shown[12] that the value of $F(A,t)$ is characterized by the following partial differential equation (PDE)

$$\begin{array}{ll} F_t + \frac{1}{2}\sigma^2 A^2 F_{AA} + (\mu-\delta) A F_A - \nu F + (\delta A - r_d D) = 0 & \text{for } A \geq A_C, \\ F_t + \frac{1}{2}\sigma^2 A^2 F_{AA} + (\mu-\delta) A F_A - \nu F + (\delta A - r_d D) - \lambda F = 0 & \text{for } A < A_C, \end{array} \tag{A1}$$

where $F_t$ denotes the first derivative of the equity value $F$ with respect to time $t$, and $F_A$ and $F_{AA}$ denote the first and second derivatives of the equity value $F$ with respect to the bank's asset value $A$. The PDE captures the dynamic of equity $F$ in the next time interval $dt$.

Since equityholders receive, or pay depending on the sign, $\delta A - r_d D$ as long as the bank is open, this term appears in both parts of the PDE (A1). When $A \geq A_C$, the bank satisfies the minimum capital requirements rule and it will not be closed in case an audit takes place. In contrast, if $A < A_C$ and the supervisor audits the bank, it will close the bank and equityholders will be expropriated of their equity, which is represented by the term $-\lambda F$ in the second part of the ODE. $\lambda$ represents the probability of an audit taking place and $F$ the loss for equityholders given the audit takes place.

We can write (A1) as

$$F_t + \frac{1}{2}\sigma^2 A^2 F_{AA} + \alpha_1 A F_A - \alpha_2 F + (\delta A - r_d D) = 0, \tag{A2}$$

[12] See, for example, Dixit and Pindyck (1994).

where:

$$\alpha_1 = \mu - \delta, \tag{A3}$$

$$\alpha_2 = \begin{cases} \nu & \text{if } A \geq A_C, \\ \nu + \lambda & \text{if } A < A_C. \end{cases} \tag{A4}$$

The boundary condition at maturity $T$ is given by

$$F(A,T) = \max\{A - D, 0\} \qquad \text{for all } A. \tag{A5}$$

We define the function $G(A,t)$ by

$$G(A,t) = F(A,t) + \frac{r_d D}{\alpha_2} + \frac{\delta A}{\alpha_1 - \alpha_2}, \tag{A6}$$

which implies

$$F_t = G_t, \tag{A7}$$

$$F_A = G_A - \frac{\delta}{\alpha_1 - \alpha_2}, \tag{A8}$$

$$F_{AA} = G_{AA}. \tag{A9}$$

The initial PDE (A2) expressed in terms of $G$ is

$$G_t + \frac{1}{2}\sigma^2 A^2 G_{AA} + \alpha_1 A G_A - \alpha_2 G = 0. \tag{A10}$$

Next, we consider the change of variable

$$x = \ln A, \tag{A11}$$

which implies

$$AG_A = G_x, \tag{A12}$$

$$A^2 G_{AA} = G_{xx} - G_x, \tag{A13}$$

transforming (A10) into

$$G_t + \left(\alpha_1 - \frac{\sigma^2}{2}\right) G_x + \frac{\sigma^2}{2} G_{xx} - \alpha_2 G = 0. \tag{A14}$$

The boundary condition (A5) becomes

$$G(A,T) = \max\{A - D, 0\} + \frac{r_d D}{\alpha_2} + \frac{\delta A}{\alpha_1 - \alpha_2}. \tag{A15}$$

We assume that at $T$ the bank can not be audited and therefore closed. Thus, $\alpha_2$ is always equal to $\nu$ at (A15) for all values of $A$ at $T$.

We solve for the value of $G$ numerically, using an explicit finite differences method.[13] We take a grid $\{x_1, ..., x_n, ..., x_N\}$ for $x$ and $\{t_1, ..., t_m, ..., t_M\}$ for $t$, where (i) $t_1 < ... < t_m < ... < t_M$, (ii) $t_{i+1} - t_i = \Delta t$ for all $i = 1, ...N-1$, (iii) $t_M = T$, (iv) $t_1 = t$, (v) $x_1 = \ln A_1$, (vi) $x_N = \ln A_N$, (vii) $x_1 < ... < x_m < ... < x_M$, (ix) $x_{i+1} - x_i = \Delta x$ for all $i = 1, ...N-1$ and (x) $g_{m,n}$ and $f_{m,n}$ represent the approximation of $G$ and $F$ at the grid point given by $(t_m, x_n)$.

Approximating the derivatives $G_t$, $G_x$ and $G_{xx}$ at the grid point $(t_m, x_n)$, (A14) becomes

$$\frac{g_{m+1,n} - g_{m,n}}{\Delta t} + \left(\alpha_1 - \frac{\sigma^2}{2}\right) \frac{g_{m+1,n+1} - g_{m+1,n-1}}{2\Delta x} + \frac{\sigma^2}{2} \frac{g_{m+1,n+1} - 2g_{m+1,n} + g_{m+1,n-1}}{(\Delta x)^2} - \alpha_2 g_{m,n} = 0. \tag{A16}$$

Rearranging, we can express $g_{m,n}$ as a function of $g_{m+1,n+1}$, $g_{m+1,n}$ and $g_{m+1,n-1}$ :

$$g_{m,n} = \frac{1}{1+\alpha_2 \Delta t} \left[p_u g_{m+1,n+1} + p_m g_{m+1,n} + p_d g_{m+1,n-1}\right], \tag{A17}$$

$$p_u = \frac{1}{2}\Delta t \left(\frac{\sigma^2}{(\Delta x)^2} + \frac{\alpha_1 - \frac{\sigma^2}{2}}{\Delta x}\right), \tag{A18}$$

$$p_m = 1 - \frac{\sigma^2 \Delta t}{(\Delta x)^2}, \tag{A19}$$

$$p_d = \frac{1}{2}\Delta t \left(\frac{\sigma^2}{(\Delta x)^2} - \frac{\alpha_1 - \frac{\sigma^2}{2}}{\Delta x}\right). \tag{A20}$$

Equation (A17) is equivalent to taking discounted expectations, where $\frac{1}{1+\alpha_2 \Delta t}$ is an approximation of $e^{-\alpha_2 \Delta t}$. Therefore the explicit finite difference method is equivalent to approximating the diffusion process by a discrete trinomial process.

Initially, the boundary condition (A15) gives us $g_{M,1}, ..., g_{M,N}$. Given $g_{m+1,1}, ..., g_{m+1,N}$ we compute $g_{m,2}, ..., g_{m,N-1}$ for $m = 1, ...M-1$, using (A17). We take a sufficiently low level for $A_0 = e^{x_0}$ and a sufficiently high level for $A_N = e^{x_N}$ and compute $g_{m,1}$ and $g_{m,N}$ as suggested by Clewlow and Strickland (1998, Section 3.3):

$$\frac{\partial F}{\partial A} = \frac{\partial G}{\partial A} - \frac{\delta}{\alpha_1 - \alpha_2} = 1 \qquad \text{for } A \text{ large}, \tag{A21}$$

$$\frac{\partial F}{\partial A} = \frac{\partial G}{\partial A} - \frac{\delta}{\alpha_1 - \alpha_2} = 0 \qquad \text{for } A \text{ small}, \tag{A22}$$

which imply

$$\frac{g_{m,N} - g_{m,N-1}}{A_N - A_{N-1}} = 1 + \frac{\delta}{\alpha_1 - \alpha_2}, \tag{A23}$$

$$\frac{g_{m,2} - g_{m,1}}{A_2 - A_1} = \frac{\delta}{\alpha_1 - \alpha_2}, \tag{A24}$$

[13] See, among others, Clewlow and Strickland (1998) and Lapidus and Pinder (1999) for details.

and therefore

$$g_{m,N} = g_{m,N-1} + \left(1 + \frac{\delta}{\alpha_1 - \alpha_2}\right)(A_N - A_{N-1}), \tag{A25}$$

$$g_{m,1} = g_{m,2} - \frac{\delta}{\alpha_1 - \alpha_2}(A_2 - A_1). \tag{A26}$$

In order for the explicit method to be consistent, stable and convergent we take $\Delta t$ and $\Delta x$ to satisfy (as proposed by Clewlow and Strickland 1998):

$$\frac{\Delta t}{(\Delta x)^2} \leq \frac{1}{3\sigma^2}. \tag{A27}$$

When (in Appendix A.2) bank's equityholders can choose between high or low risk, in order for the previous condition to hold independently of the risk level we take $\Delta t$ and $\Delta x$ to satisfy

$$\frac{\Delta t}{(\Delta x)^2} \leq \frac{1}{3\sigma_H^2}. \tag{A28}$$

Given $g_{m,n}$ and using (A6) we derive the approximation of the value of equity $F$ at each grid point $(t_m, x_n)$ :

$$f_{m,n} = g_{m,n} - \frac{r_d D}{\alpha_2} - \frac{\delta A_n}{\alpha_1 - \alpha_2}. \tag{A29}$$

## A.2 Closure and risk switching options

At each point in time, besides having the option to voluntarily close the bank, equityholders have the option of choosing one out of two different risk levels $\sigma \in \{\sigma_L, \sigma_H\}$. The probabilities $p_d$, $p_m$ and $p_u$, given by (A20), (A19) and (A18) respectively, are now risk dependent.

Consider a capital requirements (2) rule where risk weighted assets are given by (3). This closure rule implies the closure levels (4).

To compute the approximation $f_{m,n}$ of the value of equity at each grid point $(t_m, x_n)$ we proceed as follows:

1. Denote by $g_{m+1,1}, ..., g_{m+1,N}$ the approximation of $G$ at the time step $m+1$ when both closure and risk switching options are considered. For each risk level $\sigma \in \{\sigma_L, \sigma_H\}$ we compute, as in Appendix A.1, the value of equity without voluntary closure and with a fixed risk level $\sigma$. Let's call them $g(\sigma_L)_{m,1}, ..., g(\sigma_L)_{m,N}$ and $g(\sigma_H)_{m+1,1}, ..., g(\sigma_H)_{m+1,N}$.

   Note that, if $\theta > 0$, $\alpha_2$ is different for each risk level, i.e.

$$\alpha_2(\sigma) = \begin{cases} v + \lambda & \text{if } A_n < A_C(\sigma, \theta > 0), \\ v & \text{if } A_n \geq A_C(\sigma, \theta > 0). \end{cases} \tag{A30}$$

   In particular, $\alpha_2$ will be different for each risk level for grid points $(t_m, x_n)$ in which $A_C(\sigma_L, \theta > 0) < A_n < A_C(\sigma_H, \theta > 0)$.

2. Next, we take into account the risk switching option. For each risk level $\sigma \in \{\sigma_L, \sigma_H\}$, the approximation of the value of equity at $(t_m, x_n)$ when there is no risk switching option, denoted by $\hat{f}_{m,n}(\sigma)$, is given by

$$\hat{f}_{m,n}(\sigma) = g_{m,n}(\sigma) - \frac{r_d D}{\alpha_2(\sigma)} - \frac{\delta A_n}{\alpha_1 - \alpha_2(\sigma)}, \tag{A31}$$

where $g_{m,n}(\sigma)$ was computed in step 1 and $\alpha_2(\sigma)$ is given by (A30). At each grid point $(t_m, x_n)$ we compute $\hat{f}_{m,n}(\sigma_L)$ and $\hat{f}_{m,n}(\sigma_H)$ and choose the risk level $\sigma^*_{m,n}$ :

$$\sigma^*_{m,n} = \begin{cases} \sigma_L & \text{if } \hat{f}_{m,n}(\sigma_H) < \hat{f}_{m,n}(\sigma_L), \\ \sigma_H & \text{otherwise.} \end{cases} \tag{A32}$$

Therefore, the approximation of the value of equity at $(t_m, x_n)$ when there is risk switching option, denoted by $\tilde{f}_{m,n}$, is given by

$$\tilde{f}_{m,n} = \hat{f}_{m,n}\left(\sigma^*_{m,n}\right). \tag{A33}$$

3. Finally, we consider the voluntary closure option. The approximation of the value of equity at $(t_m, x_n)$ when there is risk switching and closure option, denoted by $f_{m,n}$, is given by

$$f_{m,n} = \max\left\{\tilde{f}_{m,n}, 0\right\}. \tag{A34}$$

If $m > 1$, we go back to step 1, where the approximation of $G$ for grid points at time step $m$, $g_{m,1}, ..., g_{m,N}$ is given by

$$g_{m,n} = f_{m,n} - \frac{r_d D}{\alpha_2\left(\sigma^*_{m,n}\right)} - \frac{\delta A_n}{\alpha_1 - \alpha_2\left(\sigma^*_{m,n}\right)}, \tag{A35}$$

for $n = 1, ..., N$.

# B Parameter values

We use Bhattacharya et al. (2002) calibration exercise, which uses data on commercial banks over the period 1989-98, to fix the values of the risk free interest rate $r$, payout ratio $\delta$ and high and low risk levels, $\sigma_L$ and $\sigma_H$. The Office for the Comptroller of the Currency (OCC) regulates and supervises more than 2200 national banks and 56 federal branches of foreign banks in the U.S., accounting for more than 55 percent of the total assets of all U.S. commercial banks at 2004. According to OCC (1999): "By statute, every national bank must receive a full-scope, on-site ('safety and soundness') examination not less than once every 12 months. The OCC may extend the 12-month interval to 18 months for institutions meeting certain criteria for size and condition. Most community banks qualify for the 18-month examination cycle." An average audit frequency of 15 months corresponds to an audit frequency $\lambda$ of 0.8.

| Parameter | Description | Value |
|---|---|---|
| $\delta$ | Payout ratio | 4.2% |
| $r_d$ | Deposits rate | 5% |
| $D$ | Deposits face value | 1 |
| $\mu$ | Instantaneous rate of return | 5% |
| $\nu$ | Cost of capital | 5% |
| $\lambda$ | Supervisor audit frequency | 0.8 |
| $T-t$ | Maturity (years) | 10 |
| $A_1$ | Minimum asset value | 0.5 |
| $A_N$ | Maximum asset value | 10 |
| $\sigma_L$ | Low risk level | 10% |
| $\sigma_H$ | High risk level | 20% |

Benchmark case parameter values.

The results reported in the main text do not qualitatively vary with the choice of the parameter values.

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