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Credit Derivatives

Models, Market & Products

King's College London - 2007

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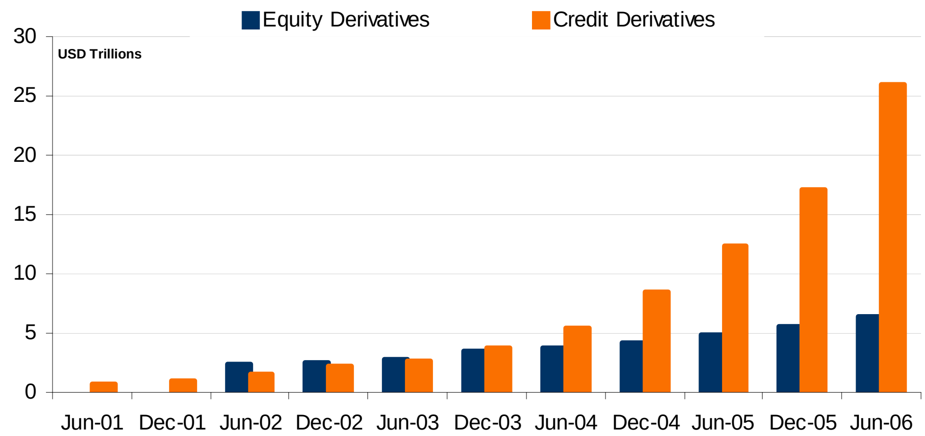
Outline

- **Lecture 1:**
  - **Credit Risk Models**
  - **Credit Default Swap Pricing**
- Lecture 2:
  - Credit Derivatives Market
  - Product Review
- Lecture 3:
  - Credit Risk Correlation Modelling
  - Pricing CDOs & Tranches

Derivatives: Credit vs. Equity

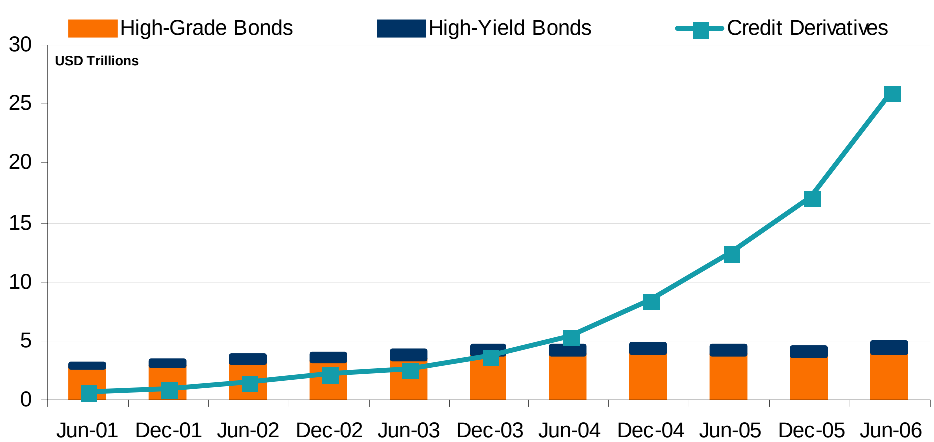
Equity derivatives don't keep pace



Source: ISDA Market Survey. $ in trn.

Cash vs. Derivatives Markets

Bonds don't keep pace either



Source: ISDA Market Survey, Merrill Lynch Index System. $ in trn.

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Credit Risk Models

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Credit Risk

- **What is credit risk?**

  Risk of not receiving an amount of money you were promised

- **Corporate Credit Risk**

  Firms borrow money from several sources:

  - From banks: **Loans**
  - Directly from investors: **Bonds**

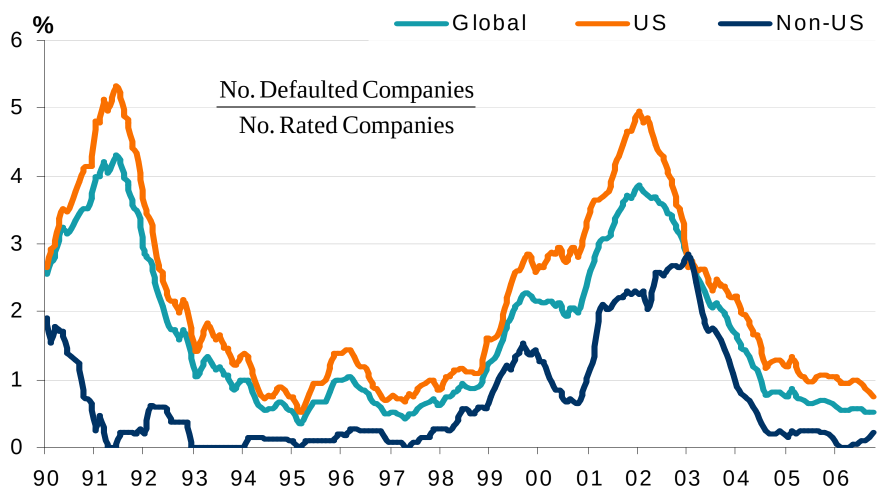
Components of Credit Risk

- **Default**
  - Not receiving ALL you were promised
  - **Probability of Default**

- **Recovery**
  - How much do you receive?
  - Fraction of the owed amount recovered by the lender
  - Usually expressed as a % of the owed notional

  - Loss Given Default = 1 – Recovery Rate

Moody's historical default rate



Source: Moody's Trailing 12-month Issuer Default Rates. In %.

What do we do with the risk?

- A firm has a given probability of defaulting and not paying all it borrowed.
- So What? **Where does credit risk show up?**
- **Credit risk will determine the borrowing costs** of that company:
  - The firm will have to pay higher coupon on its bonds in order for investors to be willing to buy them
  - Banks will require higher interests on the loan

But … **How do we translate the risk into a coupon level?**

Bonds: First Stop

Zero-Coupon Bond

Today ***t*** — Maturity ***T***

- Promised payments:
  - No coupons
  - Payment of notional ***N*** at maturity ***T*** (**if there is no default before *T***)

- If the firm defaults before maturity
  - we recover a fraction ***R*** of the notional ***N***
  - whenever the default happens

Zero-Coupon Bond Price

Today ***t*** — Maturity ***T***

Price ***P* (?)** — Notional ***N***

- What is the price **P** today?

- It depends on:
  - **Probability that the firm defaults** before maturity
  - **Recovered rate R** if there is default

Zero-Coupon Bond Price

- Let τ denote the time of default (No default: $\tau > T$)
- Let P_t denote the price, at time *t*, of a zero-coupon bond with
  - maturity *T*,
  - notional *N*,
  - Expected recovery rate *R*, independent of default probs.

- Default can only happen at certain dates $t_1, ..., t_N$
- Risk free discount factor:

  Assume a constant interest rate *r*

$$DF(t_n) = e^{-rt_n}$$

Default and Survival Probabilities

- Imagine we are at time *t* and the firm is alive
- Default Probability:

  Probability that the firm defaults by time *T*

$$P[\tau \leq T \mid \tau > t] = P[\tau \leq T]$$

- Survival Probability

  Probability that the firm does not default by time *T*

$$1 - P[\tau \leq T]$$

Assumptions

- We work under a risk neutral probability measure, equivalent to the physical probability measure, which uses the money market account as numeraire.
- Asset processes, discounted by the money market account are martingales under the risk neutral probability measure.
- Though one can relax this assumption, we assume market completeness: all assets can be hedged and priced.
- We will assume interest rates are independent of default times and recovery rates.

Zero-Coupon Bond Price

- The price has two parts:
  - Payment at maturity if there is no default
  - Payment if there is default

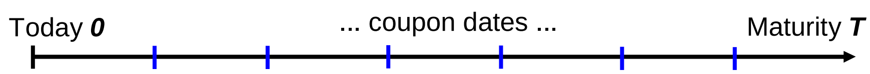
Each part "weighted" by the corresponding probability

$$P_t = E[N \cdot DF(T)] + E[(1-R) \cdot N \cdot \int_t^T DF(t) dP(\tau = t)]$$

$$= N \cdot DF(T) \cdot P(\tau > T)$$

$$+ (1-R) \cdot N \cdot \sum_{i=1}^{N} DF(t_i) \cdot [P(\tau > t_{i-1}) - P(\tau > t_i)]$$

Coupon Bond



- Promised payments:
  - Coupon ***c*** at each coupon date
  - Payment of notional ***N* + *c*** at maturity
    (**if there is no default before *T***)
- If the firm defaults before maturity $\tau < T$,
  - we recover a fraction ***R*** of the notional ***N***
  - when the default happens

Coupon Bond

- Coupon bond: combination of Zero-Coupon bonds:
  - Each coupon represents a zero-coupon bond with notional ***c***
  - The notional payment at maturity represents a zero-coupon bond with notional ***N* + *c***
- Its price is the sum of the zero-coupon bond prices

Bond Price

- Important assumptions we have made:
  - We know the expected recovery rate *R*
  - Default probability, recovery rate and discount factors are independent
  - Defaults can only happen at certain dates
- Relaxing them will complicate the pricing formula

Bond Price

- The price of a bond
  - Increases as the default probability decreases
  - Increases as the recovery amount increases
- Market jargon: The investor
  - Has a long position on the bond, i.e.
  - Credit protection seller
- The investor
  - Benefits if default probabilities go down
  - Losses money if default probabilities go up or there is a default

Bond Price

- The investor
  - Has a long position on the bond, i.e.
  - (Credit) protection seller
- **What if the investor ...**
  - Wants to make money if default probabilities go up?
  - Already has a bond/loan of a given company
    and wants to hedge it against a default or an increase in default probabilities?
  - Wants to be exposed to the credit risk of a company without having to pay the price of a bond?
- **Credit derivatives** provide answers to these questions.

Credit Risk:

Sources of Information

Where?

- Credit risk = default probabilities & recovery rate
- Where can we find information about them?

- Two main sources:
  - **Go to the firm itself** and check:
    - Value of assets
    - Balance sheets
    - Amount of debt, interest rates, …
    - Expected cash flows, …
  - **Go to the market** (reverse engineering)
    - Prices should tell us something about the firm's credit risk

Where?

- Two main sources:

  - **Go to the firm itself**
    - First hand information
    - Information about the true determinants of default and recovery rates

  - **Go to the market**
    - "Second-hand" information
    - Market's assessment about credit risk
    - Maybe "the market" has already gone to the firm, so prices should already reflect that information

Where?

- Each source of information aligns itself with a modelling approach:

  - **Structural Models: Go to the firm itself**
    – Credit risk comes from the firm's fundamentals

  - **Reduced Models: Go to the market**
    – Credit risk comes from the market assessment

- Any preferences?

Structural vs Reduced Form Models

- **Reduced form models**
  – Most popular within the industry for their easy and quick calibration to market prices
  – They have the (quasi) monopoly in CDS and bond pricing
- **Structural models**
  – More favoured by academia (finance/economics) because they try to explain things
  – Very hard to calibrate (properly).
  – In industry: the whole CDO pricing machinery relies on a simple structural model (though most people are not aware of)
  – Consistent framework to price equity-credit products

Structural Models

Structural Models

- Balance sheet of the firm

Assets	Debt
	Equity

- Debt is composed of different bonds and loans
- We have to find out firm's default probabilities and recovery rates …

  … and plug it into the formula to price bonds and credit derivatives.

Structural Models

- From simple to sophisticated
  - Merton Model (1974)
  - First Passage Models (1976)
  - Liquidation Process Models (2004)
  - State Dependent Models (2004)

- Their practical application has been limited by the model's assumptions … but …

  they are the only (?) way forward to problems such as equity-credit joint modelling

- Require extensive econometric work

Merton Model

- **Black-Scholes formula application to credit risk**

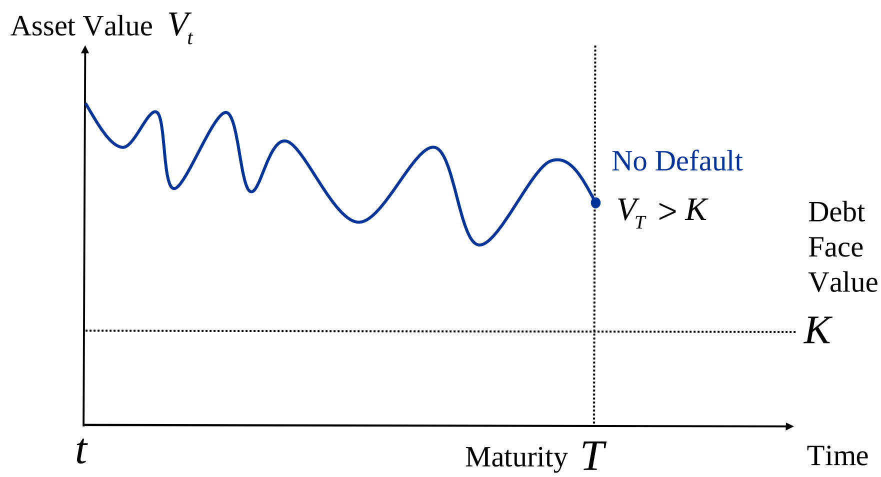
- Take the firm today at time t

Assets	Debt
	Equity

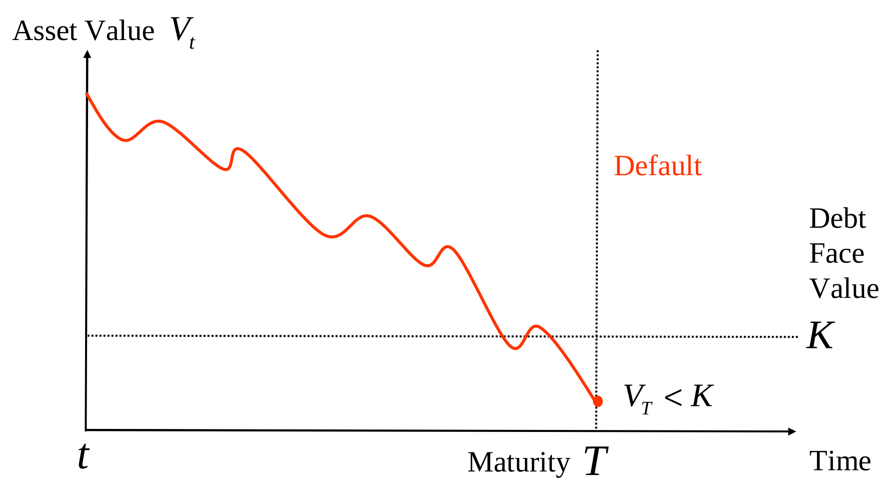
- Asset value V_t
- Assume debt is composed of a zero-coupon bond with
  - Maturity T
  - Face Value K

- Default at T iff $V_T < K$

Merton Model



Merton Model



Merton Model

- Payoffs at T
  - Debtholders $\max\{V_T, K\}$
  - Equityholders $\max\{V_T - K, 0\}$
- **Equity is a call option on the asset value** V_t with strike K
- **And debt is just assets minus equity**

Merton Model

- We can price equity using the Black-Scholes formula and debt as the difference between assets and equity
- Assumptions required:
  - Debt is just a zero-coupon bond with maturity T and face value K
  - Asset value follows a Geometric Brownian Motion

$$dV_t = rV_t dt + \sigma V_t dW_t$$

Merton Model

- Value of Equity:

$$E_t = V_t \Phi(d_1) - e^{-r(T-t)} K \Phi(d_2)$$

where $\Phi(\cdot)$ is the distribution function of a standard normal random variable.

$$d_1 = \frac{\ln\left(\frac{e^{r(T-t)} V_t}{K}\right) + \frac{1}{2}\sigma^2 (T-t)}{\sigma\sqrt{T-t}} \qquad d_2 = d_1 - \sigma\sqrt{T-t}$$

Merton Model

- Probability of default before time T

$$P[V_T < K] = \Phi(-d_2)$$

- Expected recovery rate $E_t\left[\frac{V_T}{K}\right]$

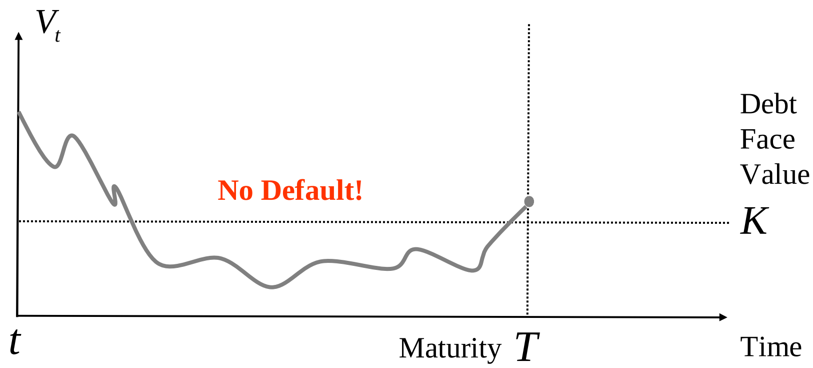
- Value of debt at time t

$$V_t - E_t$$

- That's all we wanted.

Merton Model

- Assuming that the debt is just a zero coupon bond:
  - Is not very realistic
  - Implies the firm can only default at maturity



Merton Model

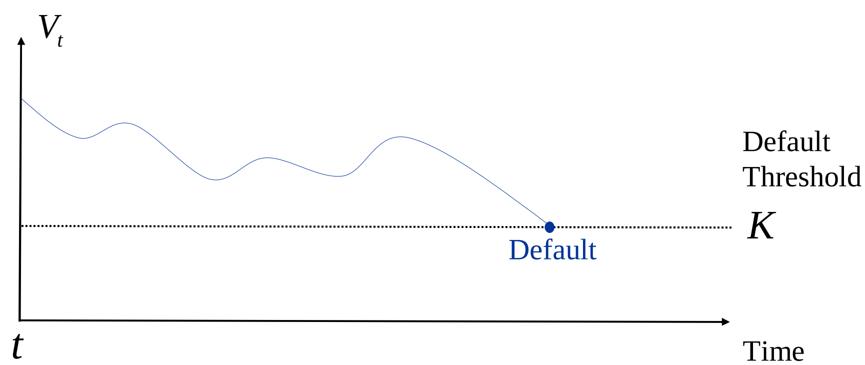
- Merton (1974) was the first structural model

- Subsequent models tackle some of its restrictive assumptions to make the model more realistic:

  - First Passage Models (Black and Cox 1976)
  - Liquidation Process Models
  - State Dependent Models

First Passage Models

- A firm defaults whenever its asset value V_t falls below a lower barrier K
- Default can take place at any time



First Passage Models

- First introduced by Black and Cox (1978)
  (Closed form solutions using barrier options formulae)
- Extended to account for:
  - Stochastic interest rates
  - Bankruptcy costs
  - Taxes
  - Debt Subordination
  - Strategic Default
  - Jumps in the asset value process
  - …

Reduced Form Models

Reduced form models

- Reduced form models rely on the market as a source of credit risk information
- Different modelling approaches consistent with a reduced form model:
  - **Intensity Models are the most popular**
  - Brody, Hughston and Macrina (2006):
    - Based on the amount and precision of the information received by market participants, through market prices, about the firm's credit quality.
    - Does not require intensities. Work in progress.
- Intensity models are the most well known among investment banks and financial mathematics academia

Intensity Models

- Probability background
  - A Poisson random variable measures a variable which takes values 0, 1, 2, …
  - The starting value is 0
  - Examples:
    - Number of times it rains in London
    - Number of times a firm/country defaults (?)

- A Poisson variable is characterized by an "intensity" or hazard rate, which determines the likelihood of each jump
- **In intensity models, default time is modelled as the first jump of a Poisson process with a given intensity**

Intensity Models

- Imagine the default of a firm is modelled using a Poisson process
  - Default time τ
  - Intensity:

$$\lambda_t = \lim_{h \to 0} \frac{P[\tau \in (t, t+h] \mid \tau > t]}{h}$$

Default intensity (or hazard rate):

Conditional (on the firm being alive at *t*) instantaneous default probability

Intensity Models

- **Default can take place at any time** with a given probability
- Intensity models provide **simple formulas**

  – Survival probability from *t* to *T*

$$s(t,T) = P[\tau > T \mid \tau > t] = E[e^{\int_t^T \lambda_s ds}]$$

  – Default probability

$$P[\tau \leq T \mid \tau > t] = 1 - s(t,T)$$

Intensity Models

- Simplest case: constant intensity
  – Default intensity is the same, no matter we are in a recession or in a boom
  – Simple maths

$$P[\tau \leq T \mid \tau > t] = 1 - e^{-(T-t)\lambda}$$

- Other options:
  – Time dependent
  – Stochastic:
    - Single factor
    - Multi-factor to capture correlations

Zero-Coupon Bond Price

- The price has two parts:

  – Payment at maturity if there is no default
  – Payment if there is default

Each part "weighted" by the corresponding probability

$$P_t = E[N \cdot DF(T)] + E[(1-R) \cdot N \cdot \int_t^T DF(t) dP(\tau = t)]$$

$$= N \cdot DF(T) \cdot P(\tau > T)$$

$$+ (1-R) \cdot N \cdot \sum_{i=1}^{N} DF(t_i) \cdot [P(\tau > t_{i-1}) - P(\tau > t_i)]$$

Calibrating Intensity Models

- Assume a constant intensity & zero expected recovery

$$P_t = N \cdot DF(T) \cdot e^{-(T-t)\lambda}$$

- If we know the price of the bond we can derive the default intensity

- And we can use it to price:
  – Other bonds
  – CDS (Elizalde 2005: Spanish firms)

Sovereign Defaults

- In intensity models, default is determined by the **FIRST** jump of a Poisson random variable with a given intensity.

Country	Year	Recovery	Volume
Pakistan	1998	65%	750
Russia	1998	18%	73336
Ukraine	1998	-	1,422
Venezuela	1998	-	270
Ecuador	**1999**	**44%**	**6,603**
Peru	2000	-	4,870
Ukraine	2000	69%	1,063
Ivory Coast	2000	18%	-
Argentina	2001	33%	82,268
Moldova	2001	66%	145
Uruguay	2003	66%	550
Grenada	2004	65%	-
Dominican	2005	92%	1,100

Source: Moody's. Volume in USD millions.

Can a Firm Default at any Time?

Default at any time?

- Intensity models assume you can
- Some structural models do (e.g. with jumps)

- If a firm does not have any coupon payment tomorrow, Can it default?

- How do you define default?
  - The firm can not miss a payment tomorrow
  - But it can file for bankruptcy, or it can reorganize its debt, …
- First define what “default” is, then answer the question

Reconciliation

Information

- Several papers have bridged the gap between reduced and structural models
- Based on incomplete information versions of structural models
  - they obtain reduced form models
  - in which the intensity of default is not given exogenously
  - but determined endogenously within the model
  - and it is a function of the firm's characteristics and the level of information investors have
- Key element: How much information investors have

How much information

- **"Structural models are based on the information set available to the firm's management, which includes continuous-time observations of both asset values and liabilities.**
- **Reduced-form models are based on the information set available to the market, typically including only partial observations of both the firm's asset values and liabilities.**
- […] it is possible to transform a structural model with a predictable default time into a reduced-form model, with a totally inaccessible default time, by altering the information sets available for modelling purposes"

Guo, Jarrow and Zeng (2005, p.2)

Which model would you use?

Which model would you use?

- A reduced one?
  - Calibration is “simple” to market prices
  - Can be used for hedging purposes and to price other instruments
  - But you can not beat the market (or can you?)

- A structural one?
  - Calibration is complex and not always successful
  - But you have a different point of view than the market
  - And you can use it to price both equity and credit products

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Credit Default Swaps

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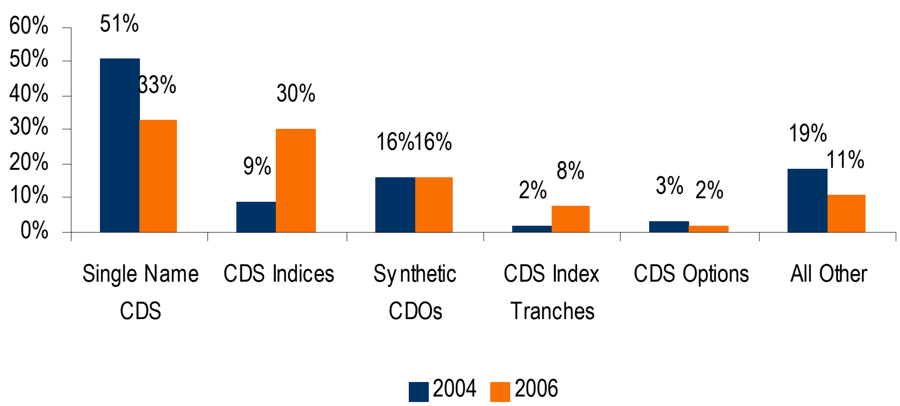
Credit Derivatives Market

CDS: Half of the market in 2004, One-third in 2006



Source: BBA 2006 Credit Derivatives Survey.

Credit Default Swap (CDS)

- Initial rationale:
  - You have a bond of company XYZ.
  - If the company defaults you will suffer losses
  - Imagine you want to hedge that risk (XYZ credit risk)
  - You have two options:
    - Sell the bond
    - Buy a financial instrument which would compensate you for the bond losses if the default occurs

- **A CDS is a financial instrument whose payoffs are tied up to the default status of a given company over a specified maturity.**

Credit Default Swap

- You need a contract which pays you if there is a default, and which pays you the losses on your bond in that case
  - Losses on the bond if there is a default = 100% - Recovery Rate

- You need somebody else agreeing to get into that contract
- That somebody else wants something in exchange
  - Let's say a yearly payment of 2% (of the bond notional)

- That "something in exchange" is what is referred to as the **"Price" or "Premium" of the CDS**

CDS: Easy I

- Contract between two counterparties:
  - Protection seller
  - Protection buyer
- The contract specifies:
  - Maturity
  - Size (notional)
  - Company and bond the contract refers to
  - Contract “**premium**”
- The payments that each party will have to make to the other during its life depend on whether a reference company has defaulted or not

CDS: Easy II

- Assume:
  - 5 year maturity
  - Underlying company XYZ with only one bond issued
  - Notional: 1.000.000 & Premium: 2%
- **Payments from the prot. seller to the prot. buyer**
  - None if the firm does not default in the next 5 years
  - If the firm defaults within 5 years:

    Payment of:

    1.000.000 * (100% - Recovery Rate Bond)

    right after the default

CDS: Easy III

- **Payments from the prot. buyer to the prot. seller**
  - Each year, a payment equal to:

    1.000.000 * 2% = 20.000

    until the first of:
    - Maturity (5 years)
    - Default of the company
  - In practice, that payment is made quarterly (5.000 per quarter).

CDS: Easy IV

- If there is no default before maturity:
  - The protection seller would pay nothing and receive 5.000 each quarter from the protection buyer
- If there is default before maturity, let's say at year 2
  - From today to year 2:
    - The protection seller would pay nothing and receive 5.000 each quarter from the protection buyer
  - At year 2 (assuming 40% recovery rate on the bond)
    - The protection seller would pay 600.000 to the protection buyer
  - The contract would terminate at that point

Credit Default Swaps (CDS)

The Bloomberg™ Definition:

- "A credit derivative transaction in which two parties enter into an agreement,…"
- "… where [**protection buyer**] pays the [**protection seller**] a fixed periodic coupon for the specified life of the agreement.."
- "…[t]he [**protection seller**] makes no payment unless a **credit event**, relating to a predetermined reference asset, occurs."
- "If such an event occurs, the [**protection seller**] will then make a payment to the first party, and the swap will terminate."

CDS Pricing

What determines the premium?

- Main risk factors:
  - **Default probabilities**
  - **Recovery rates**
- Where do you get default probabilities and recovery rates from?
- If you have the firm's default probabilities and expected recovery rate, Could you find the premium? How?
- Even if you don't know the exact premium:
  - How do default probabilities affect the premium?
  - And recovery rates?

CDS Premium

- **A CDS is a contract with two "legs"**:
  - Payments by the protection buyer
  - Payments by the protection seller
- The payments from the protection seller depend on whether there is a default (default probabilities) and on the recovery rate
- The payments from the protection buyer depend on whether there is a default (default probabilities) and on the CDS premium
- **If we know the default probabilities and recovery rates, How do you find a "fair" premium?**

CDS: Pricing

- **Premium Leg**:
  - Expected discounted value of payments by the prot. buyer
- **Default Leg**:
  - Expected discounted value of payments by the prot. Seller

- "Fair" price of a CDS:
  - **Premium** which makes
  - **Premium Leg = Default Leg**

CDS: Default Leg

- Expected discounted value of payments by the prot. seller

- The protection seller pays:
  - Loss given default $(1-R)$
  - At the time of default τ (**if any before maturity** T)

- Assume default can only happen at certain dates $t_1,...,t_N$

$$DL = (1-R)\cdot\sum_{i=1}^{N} DF(t_i)\cdot\left[P(\tau \le t_i) - P(\tau \le t_{i-1})\right]$$

CDS: Premium Leg

- Expected discounted value of payments by the prot. buyer
- The protection seller pays:
  - The premium (coupon) *c*
  - At any coupon date $t_1,\ldots,t_N$
  - Until maturity τ or default T (whichever first)

$$PL = \sum_{i=1}^{N} c \cdot DF(t_i) \cdot \left[1 - P(\tau \leq t_i)\right]$$

CDS: Premium

- By imposing the **equality between the two legs**, we can solve for the premium

$$c = \frac{(1-R) \cdot \sum_{i=1}^{N} DF(t_i) \cdot \left[P(\tau \leq t_i) - P(\tau \leq t_{i-1})\right]}{\sum_{i=1}^{N} DF(t_i) \cdot \left[1 - P(\tau \leq t_i)\right]}$$

- **Positive function of default probabilities**
- **Negative function of recovery rate**

Inputs: Default Probability & Recovery

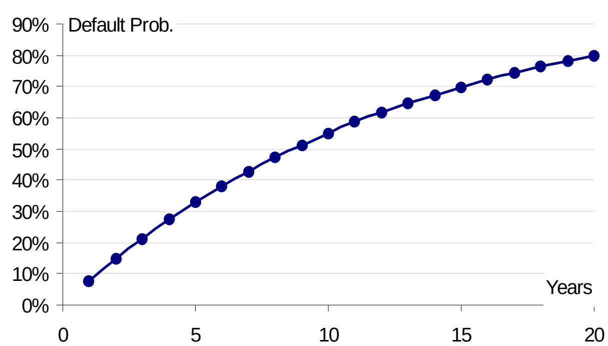
- Pricing a CDS requires an estimate of the firm's default probabilities and of the bond recovery rate

- Alternatives:
  - Use a structural model to derive both and then use it to price a CDS.
  - Assume a recovery rate for the bond, and use an intensity model to estimate a default intensity from the bond price. With such an intensity you can obtaind default probabilities. Then use them to price a CDS.

Example

- Let us assume the default probabilities of a firm are characterized by a constant intensity $\lambda = 8\%$ (e.g. we derived it from bond prices)

$$P[\tau > t] = 1 - e^{-\lambda t}$$



Example

- Let us assume the default probabilities of a firm are characterized by a constant intensity $\lambda = 8\%$
- 5% constant interest rate r
- 40% recovery rate
- 5 years maturity

$$c = \frac{(1-R)\cdot\sum_{i=1}^{N} DF(t_i)\cdot\left[P(\tau \le t_i) - P(\tau \le t_{i-1})\right]}{\sum_{i=1}^{N} DF(t_i)\cdot\left[1 - P(\tau \le t_i)\right]}$$

- CDS "fair" premium: 5%

Default probabilities and recovery rate

- **Higher default probabilities**
  (i.e. intensities if we use an intensity model)

  There is a higher probability of the protection seller having to pay the losses given default

  The protection seller will demand a higher price

  Higher CDS premium

- Recovery rate?

Inputs: Recovery & CDS premium

- In practice, CDS market prices are used to derive default probabilities, rather than the other way around.
- Assume a recovery rate 40%.
- Assume an intensity model with constant intensity λ , which implies the following default probabilities

$$P[\tau \leq t_i] = 1 - e^{-t_i \lambda}$$

- We know the market price of a CDS, c, and assume a recovery rate, e.g. 40%, we can use the CDS pricing formula to determine the unique intensity λ which is consistent with the CDS price.

Inputs: Recovery & CDS premium

- Provided discount factors, *c* and *R*, we can find λ

$$c = \frac{(1-R) \cdot \sum_{i=1}^{N} DF(t_i) \cdot \left[e^{-t_{i-1}\lambda} - e^{-t_i \lambda} \right]}{\sum_{i=1}^{N} DF(t_i) \cdot e^{-t_i \lambda}}$$

- Obviously, if *c=5%, R=40%, r=5%,* ... then ...

Inputs: Recovery & CDS premium

- The default intensity λ we find is the one the "market" is assuming ("Risk Neutral")

Will it be the same as the "real" (or "physical") one?

- We can use it to:
  - Price CDS with different maturities
  - Price bonds
  - Price other credit derivatives whose pricing formulas depend on the default probabilities of that (and other) firms

    E.g.: CDOs and Tranches.

More complex than this

- We have reviewed a simple example of how a CDS works and derived a simple formula using some "approximations" and assumptions.

- **Which approximations and assumptions we used?**

- CDS, and pricing formulas, are slightly more complex than the ones we reviewed here
- But without understanding the simple ones, you will not understand the difficult ones.

CDS: Synthetic Credit Risk

- If you have a bond you can buy protection on a CDS to hedge the bond's credit risk
- But, ... **you do not need to have the bond to enter into a CDS transaction**:
  - You can buy protection on a firm so you will profit if:
    - The firm defaults: How?
    - Default probabilities go up: How?
  - Or you can sell protection and "bet" that the firm will not default or that its default probabilities will go down
- Nowadays, most people trade CDS without holding the underlying bonds

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Credit Derivatives

Models, Market & Products

King's College London - 2007

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Outline

- Lecture 1:
  - Credit Risk Models
  - Credit Default Swap Pricing
- **Lecture 2:**
  - **Credit Derivatives Market**
  - **Product Review (CDS, LCDS, Indices)**
- Lecture 3:
  - Credit Risk Correlation Modelling
  - Pricing CDOs & Tranches

Credit Derivatives

The Bloomberg™ Definition:

- "An OTC derivative designed to **transfer credit risk** from one party to another."
- "By **synthetically** creating or eliminating credit exposures, they allow institutions to more effectively manage credit risks."
- "Most credit derivatives entail **two sources of credit exposure**…"
  - The Reference Asset: The Credit Risk Being Transferred.
  - The Counterparty: Seller or Buyer Transferring the Credit Risk.
- "**Taking many forms**, some of the more popular structures for credit derivative products include credit default swaps,…"

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Credit Derivatives Market (Overview)

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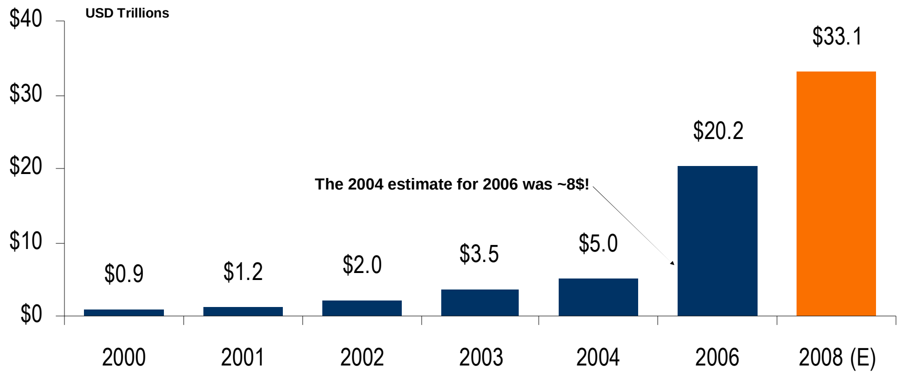
Credit Derivatives Market Growth

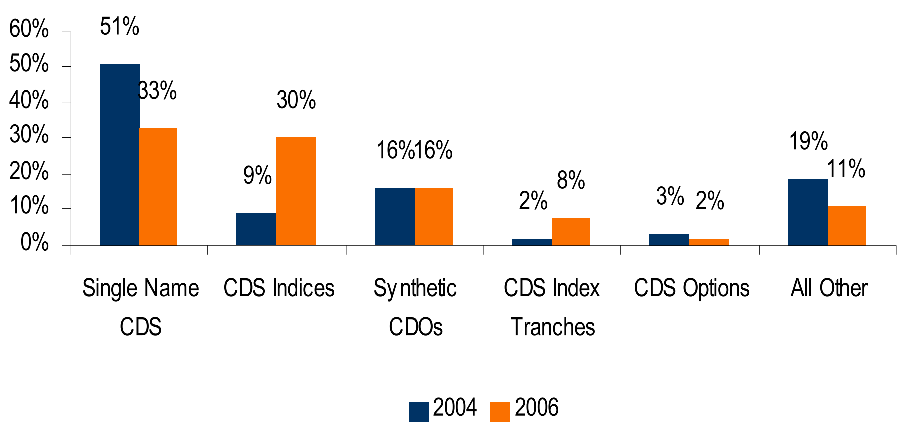
Market Grown 4x Since 2004, +50% Projected Growth to 2008



Source: BBA 2006 Credit Derivatives Survey. $ in trn.

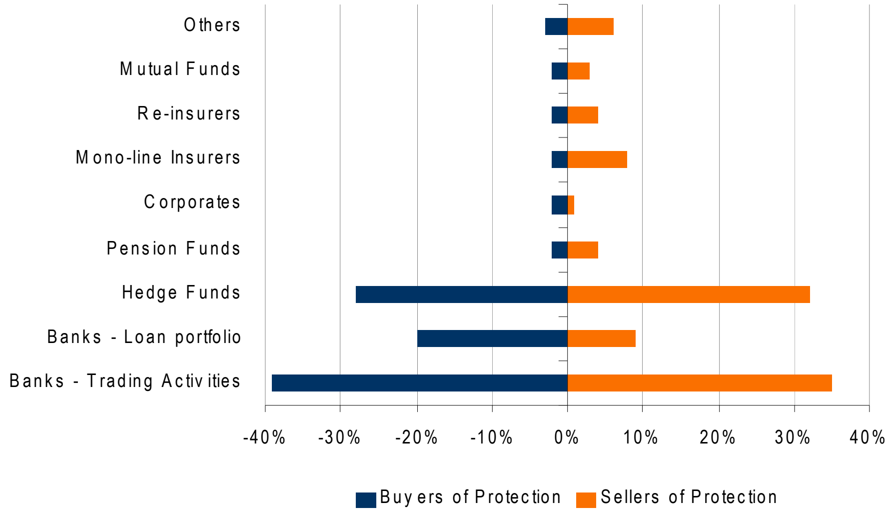
Credit Derivatives Market: Products

Indices & Tranches Have Captured Large Market Share Gains



Source: BBA 2006 Credit Derivatives Survey.

Market Players: Who's Who?



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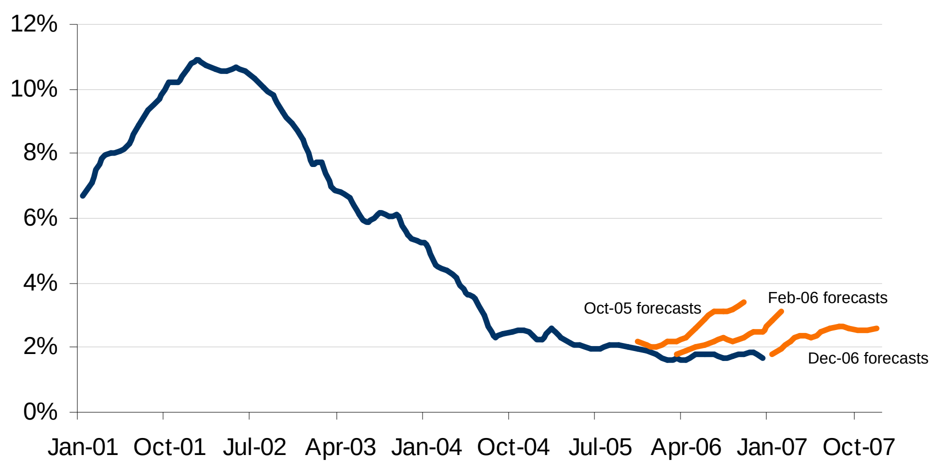
Source: BBA 2006 Credit Derivatives Survey.

7

Moody's global speculative grade default rate

Spreads seem too tight and default rates aren't rising



www.abelelizalde.com – KCL 2007 – Credit Derivatives

Source: Moody's.

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CDS:

Not that straightforward

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CDS: Easy I

- Contract between two counterparties:
  - Protection seller
  - Protection buyer
- The contract specifies:
  - Maturity
  - Size (notional)
  - Company and bond the contract refers to
  - Contract "**premium**"
- The payments that each party will have to make to the other during its life depend on whether a reference company has defaulted or not

CDS: Easy II

- Assume:
  - 5 year maturity
  - Underlying company XYZ
  - Notional: 1.000.000 & Premium: 2%
- **Payments from the prot. seller to the prot. buyer**
  - None if the firm does not default in the next 5 years
  - If the firm defaults within 5 years:
    Payment of:
    1.000.000 * (100% - Recovery Rate)
    right after the default

(assuming cash settlement)

CDS: Easy III

- **Payments from the prot. buyer to the prot. seller**
  - Each year, a payment equal to:
    1.000.000 * 2% = 20.000
    until the first of:
    - Maturity (5 years)
    - Default of the company
  - In practice, that payment is made quarterly (5.000 per quarter).

CDS: Easy IV

- If there is no default before maturity:
  - The protection seller would pay nothing and receive 5.000 each quarter from the protection buyer
- If there is default before maturity, let's say at year 2
  - From today to year 2:
    - The protection seller would pay nothing and receive 5.000 each quarter from the protection buyer
  - At year 2 (assuming 40% recovery rate on the bond)
    - The protection seller would pay 600.000 to the protection buyer
  - The contract would terminate at that point

CDS: Not that straightforward

- CDS are "**Over The Counter**" (OTC) contracts
  - Private contracts between two parties
  - Who can write the contract terms as they please
  - Who are subject to the credit risk of the other
- In general, everybody uses the same "standard" contract
  - Drafted by ISDA (International Swap and Deriv. Assoc.)
  - Increases the **liquidity** of the product
- It is worth diving into the contract details to see how they work in practice

CDS Settlement

- Settlement referts to payments upon default
- There are two different ways of "settling" a CDS contract in case of default

  (Remember, in case of default the prot. seller has to compensate the prot. buyer by the **losses due to the default**)

  - **Cash**:
    Prot. seller pays, in cash, Notional * (1 – recovery rate)

  - **Physical**:
    Prot. seller pays Notional
    Prot. buyer delivers to prot. seller "bonds" with notional value equal to CDS notional

Example CDS contract

- **Counterparties** : Seller and Buyer (of protection)
- **Maturity** : 10 years
- **Notional Amount** : £1 million
- **Reference Entity** : XYZ Ltd.
- **Reference Obligation** : Specific Bond: Coupon 5%
  Maturity 25 May 2020
- **Buyer's Payments** : 3% per annum
- **Credit Events** : Bankruptcy, Failure to Pay, Restructuring
- **Credit Event Payment**:

  (Physical) Seller pays £1 million
  Buyer delivers the Notional Amount of Deliverable Obligations

Key CDS Terms

- **Reference Entity**: The company or other legal entity to which the default swap applies.
- **Credit Event**: The "events" that trigger an exercise under contract.
- **Obligations**: The obligations of a Reference Entity with respect to which a credit event must occur for the CDS to become exercisable.
- **Deliverable Obligations**: The obligations of a Reference Entity which may be delivered by the buyer of protection if a credit event has caused the CDS to become exercisable.
- **Reference Obligation**: Specified debt obligation; always deliverable and establishes ranking in capital structure of Obligations and/or Deliverable Obligations.

Credit Default Swaps and ISDA

- Every credit swap is "documented" in a contract based on the ISDA format - called the Confirmation
- The terms used in the Confirmation are defined in the 2003 ISDA Credit Derivatives Definitions
- High level of standardization of documentation exists
- Standardization makes credit swaps easier to trade, creates transparency and facilitates market participation
- By using the market standard participants increase liquidity and easily hedge a transaction to lock in spread differential

Unwinding Transactions

- **If you have a CDS position, How do you get out?**
- Three ways:
  - Go back to original counterparty to unwind
    - No future cash flows & legal risk
  - Assignment to other counterparty
    - A new counterparty "replaces" the investor in the CDS
  - Enter into exactly offsetting transaction
    - You are a buyer of protection with one counterparty and a seller with another
    - Counterparty credit risk

Counterparty Risk

- **Risk for protection seller:**
  - Protection buyer fails to pay premium
    - Loss if CDS had positive MTM (i.e. credit tightened)
    - Claim losses to protection buyer (ISDA Master Agreement)
- **Risks for protection buyer:**
  - Reference entity defaults and seller does not pay notional amount
    - Claim losses like a senior unsecured creditor
    - Loss: 1 – (recovery of ref. entity + recovery of seller)
  - Reference entity does not default and seller does
    - Loss if CDS had positive MTM (i.e. credit widened)
    - Claim losses to protection buyer (ISDA Master Agreement)

Counterparty Risk

- What would you have to include in the CDS pricing formula to fully reflect counterparty risk?
  - Default probability of the reference entity
  - Expected recovery rate of the reference entity
  - Default probability of the protection seller
  - Expected recovery rate of the protection seller
  - Default probability of the protection buyer
  - Expected recovery rate of the protection buyer

And the correlation between all of the above.

Settlement: Physical vs. Cash

- Standard documentation includes physical settlement
  - Fears regarding physical settlement after a large default
  - Physical settlement for Delphi (Oct'05): smoother than many predicted
  - Dealers have become experienced:
    - Exercise sell contracts immediately after the default
    - Use bonds they receive to settle their buy contracts

- If **both** parties agree, the physical settlement is overridden by a cash settlement
  - After Dura defaulted (Oct'06) many contracts were cash-settled

Cash Settlement

- The protection seller pays, in cash, to the protection buyer:

(1 – Recovery) * CDS Notional

- **How do we come up with the recovery?**

Cash Settlement

- **How do we come up with the recovery?**
  - Market participants get together at some point after the default (1 month usually) and "have a look" a what is the market price of the CDS reference obligation at that point
  - They take an average of the reference obligation prices (which can be different depending on who do you ask)
  - That average is used to compute the recovery rate
  - The way this process is conducted is very specific and detailed in the contracts and ISDA protocols

- No "Delivery Option" value in cash settlement

"Exotic" CDS

- If plain vanilla CDS were not enough the market offers several variations of them:
  - Fixed recovery CDS
  - Forward starting CDS
  - Constant Maturity CDS
  - Recovery locks (Pure recovery rate trading)
  - …
- The pricing of these products builds on the pricing of vanilla CDS

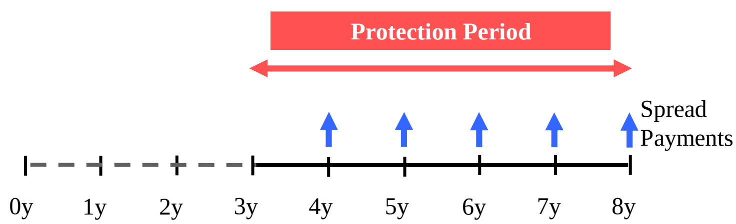
Fixed Recovery CDS

- The losses given default depend on the recovery rate. But, **realized recovery rates are not known a priory**
- A "standard" CDS is subject to two risks:
  - Default risk
  - Recovery risk
- **Fixed Recovery CDS**
  - **Fix** (at inception) **recovery rate used for setting future defaults**
  - **Eliminates recovery risk**
  - Attractive to investors with a clear view on default probabilities but not necessarily on recovery rates

Forward CDS

- A Forward CDS is an agreement to buy or sell protection for a specified period of time in the future, at a rate that is determined today, but payable only during the forward protection period.

- Example: 3x5 Forward CDS



Constant Maturity CDS

- In a vanilla CDS the premium is constant through the life of the contract
- A CMCDS is similar to a vanilla CDS except that **in a CMCDS the premium is not fixed, but resets periodically**
  - CMCDS premium is determined by a reference constant maturity (e.g. 5y) that is reset periodically (e.g. every quarter)
  - Example: CMCDS premium equals 80% of the 5y CDS premium at each quarter
  - CMCDS premium is determined as a factor of the prevailing (say) 5y CDS

Constant Maturity CDS

- CMCDS is traded in terms of the multiplication factor
  - Example CMCDS quote: 80%/90%
    - Premium for the protection seller is reset to 80% of 5y CDS
    - Premium for the protection buyer is reset to 90% of 5y CDS

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Loan (L)CDS:

Last to appear

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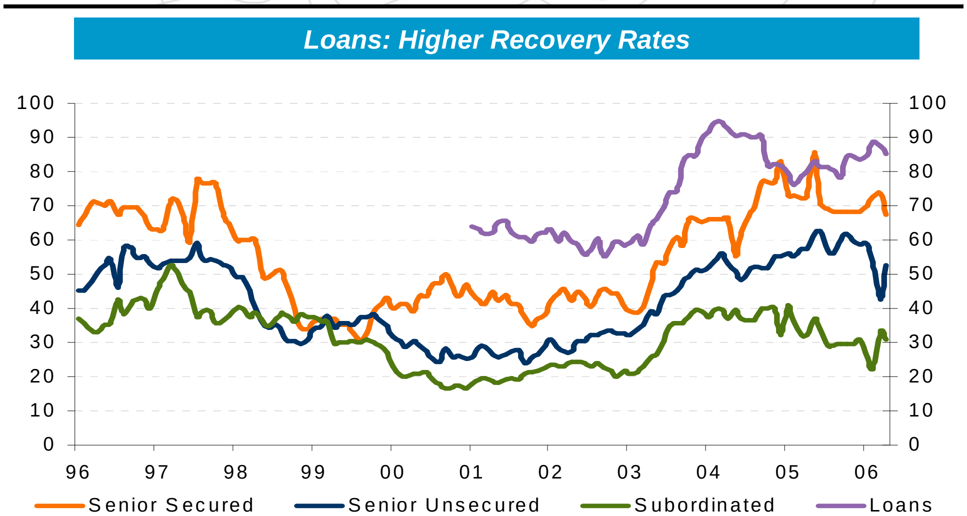
Bonds vs. Loans

- Up to 2006, CDS only referred to bonds, not to loans.
- **In Europe, the (corporate) loan market is bigger than the HY bond market.**

- Main differences between bonds and loans:
  - ***Loans are senior: if there is a default, loan holders are paid before bond holders***
  - Bonds are "public", loans not always
  - Loans are bilateral contracts between the firm and the bank
  - Bonds secondary market is more liquid

Moody's Average Recovery Rates



Source: Moody's.

Bonds vs. Loans

- Loans are typically issued by mid-low rated companies
- These companies tend to fund themselves using mostly loans
- Only banks can generally lend them
- Therefore:
  - Banks have an over-exposure to these companies
  - Other investors find it very difficult to lend (i.e. invest) in them

- LCDS market tries to address those issues

Bonds vs. Loans

- With a LCDS
  - A bank which has too much exposure to a given company credit risk (via loans) can hegde part of that risk by buying LCDS protection without having to sell the loans (which is not always feasible)
  - An external investor (e.g. hedge fund) can have access to the credit risk of that company

- The European LCDS market has been put forward with the above objectives in mind

- Commercial banks tend to be protection buyers
- Protection sellers include: hegde funds, investment banks, ...

Trading Seniority Recovery Rate

- Imagine a firm which has both loans and bonds
- Imagine there are both CDS and LCDS for that firm

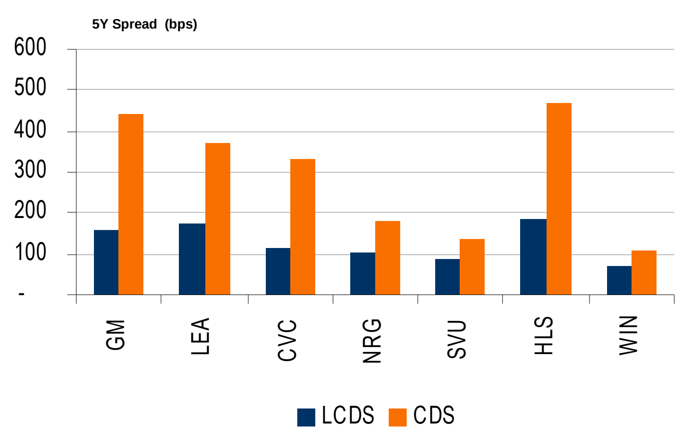
- Main risk factors affecting CDS and LCDS pricing:
  - Default probability
  - Recovery rate

- The default probability should be the same, so CDS and LCDS prices should only differ in the recovery rate the market is "assuming" for each of them

- Therefore buying CDS and selling LCDS allows one to take a view on their relative recovery rate

LCDS vs. CDS: U.S.

CDS Trades About 150 bps, or 1.5x-2.0x Wider than LCDS



Source: Bloomberg, Mark-It.

Not that simple …

- Again, contractual issues complicate the pricing of LCDS
- "Traditional" CDS:
  - You buy 5Y protection on a company @5%
  - In year 2 the company decides to take all its debt out of the market and operate as a debt-free company
  - What happens with your CDS?
  - Nothing, you have to keep paying 5% each year
    - Note that a debt free company can not default, Can it?
    - So the protection buyer is not that happy
  - Therefore, if the debt is refinanced, the CDS is not "cancelled".
  - This is one example of "succession event".

… read the LCDS contract

- In Europe, the LCDS market has been pushed forward by banks who wanted LCDS to hedge their loan exposures
- Many loans tend to be refinanced: the "official" maturity of a loan can be 10 years but the average loan life is around 4 years
- Why? If the credit quality improves the company can get a lower interest in a new loan
- Therefore if a loan is refinanced the bank does not have any credit exposure left ...

  ... and they did not want to keep paying the LCDS protection

… US or European LCDS?

- In contrast to CDS contracts, in Europe, LCDS contracts are “Cancellable”
- European LCDS contracts terminate upon the firm refinancing the loan
- In the US however, LCDS contracts are “Non-Cancellable”, like traditional CDS
- ... And in Europe one can actually find both, cancellable and non-cancellable LCDS for the same firm
- What does all of this imply for pricing?

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CDS Indices:

Second most traded product

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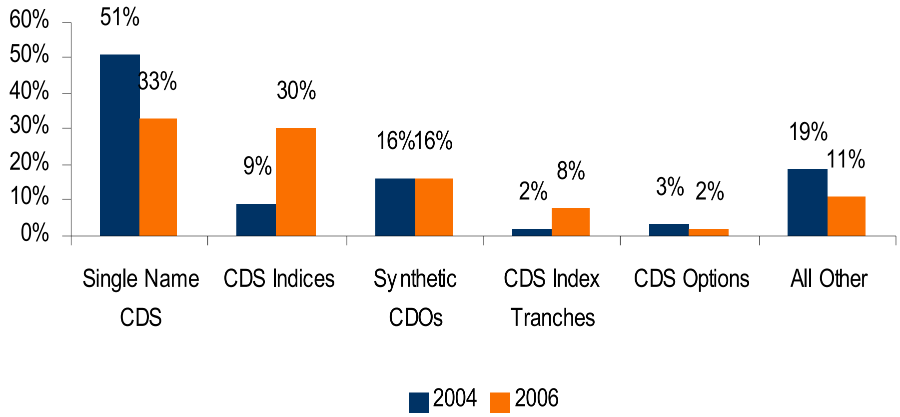
Credit Derivatives Market: Products

Index Trading: From 9% to 30% of the Market



Source: BBA 2006 Credit Derivatives Survey.

What is a CDS Index?

- What is an equity index? S&P? Dow Jones?

- The same concept applies to CDS
  - A CDS Index refers to a group of companies
  - It provides the same credit exposure as buying the CDS of each company
  - They are usually “equally” weighted in the sense that the index has the same exposure to all companies

Synthetic exposure

- A CDS contract is "synthetic" in the sense that it refers to the credit risk of a company but you do not need to buy or sell bonds or loans of that company
- In the same way, an option on Dow Jones is also synthetic
- A CDS Index is also synthetic
  - It is an **OTC contract** between two parties
  - One selling protection and the other one buying protection

Why bother?

- Why bother with indices if you can create the same exposure with single name CDS?

- Imagine you want to "sell protection" (i.e. bet that credit risk will go down) on "European Investment Grade Credit"
  - There is a huge number of European Investment Grade Companies out there. You would have to sell protection on each one and pay the bid-ask (commission(?)) on each of them
  - There is also an Index, called "iTraxx Europe" which refers to the most liquid European Investment Grade Credits

Who chooses the Index Credits?

- Take iTraxx Europe (CDS Index of IG Credits; 125 Credits)
- Every 6 months, dealers (Inv. Banks) get together and decide the index components based on:
  - They have to be European
  - They have to be Investment Grade
  - They have to have CDS traded
  - They choose the 125 most liquid credits

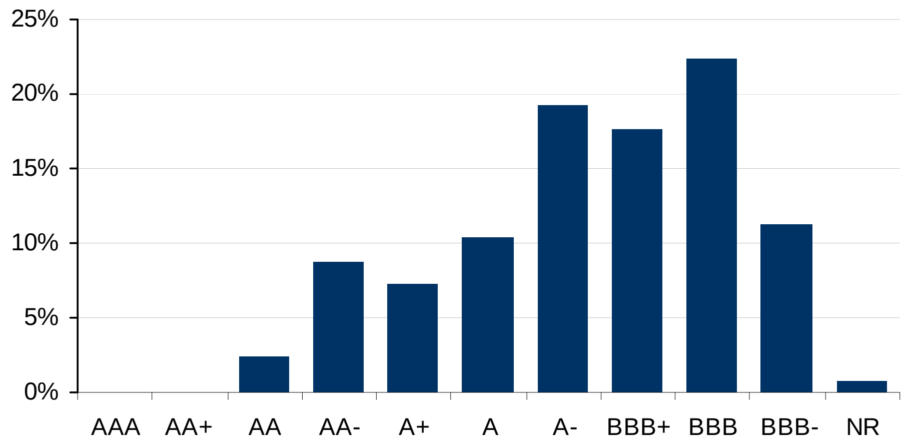
iTraxx Europe

- 125 Investment Grade European Credits
- Equally weighted portfolio (0.8% each credit)

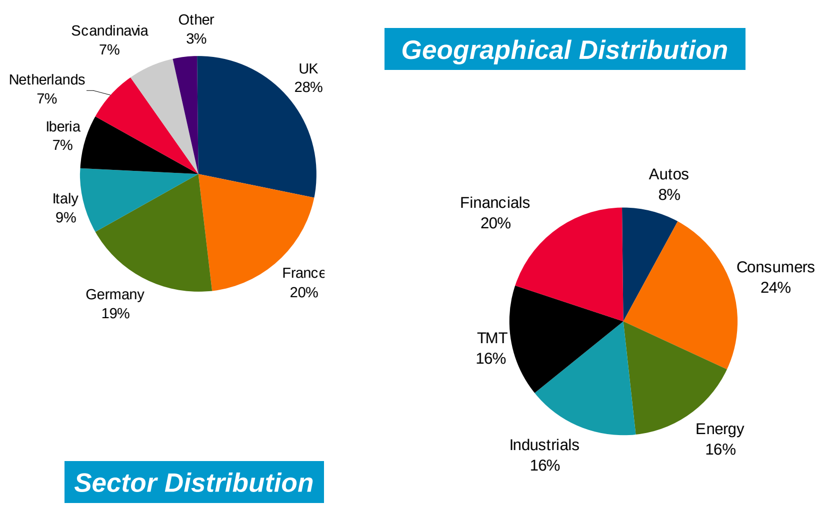
- Index composition reviewed every 6 months:
  - 20 March & 20 September
  - Liquidity: Dealers Polls
  - European, Rating (Investment Grade)
  - Sector balanced

    (10 Autos, 30 Consumers, 20 Energy, 20 Industrials, 20 TMT, 25 Financials.)

Rating Distribution: iTraxx Europe

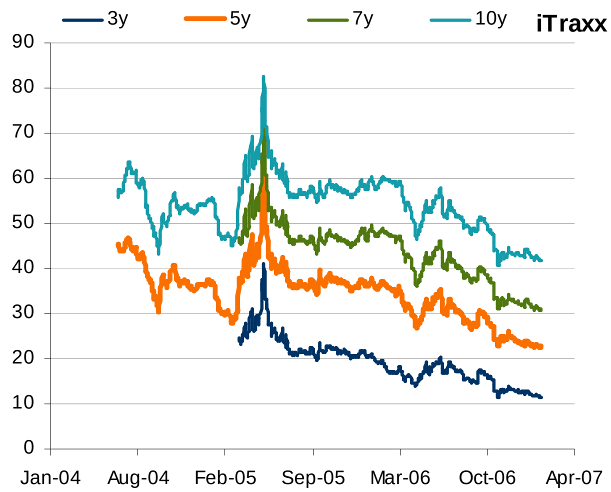


Geographical & Sector Distribution



On The Run iTraxx Europe: Historical Spreads

- 4 different maturities
- Index has a price in the form of a spread or premium (as a normal CDS)
- You pay the spread on the total notional contracted
- Spread in bps
  (1bps = 0.01%)



(Is not as easy, but this is enough)

Source: Bloomberg.

Index Mechanics

- Cash flows for a CDS Index protection seller:
  - Negative:
    - **Losses implied by each credit default**
      (e.g. iTraxx Europe: (1/125) x (1-Recovery) x Index Notional)
    - **Paid at the time of the default**
  - Positive:
    - **Quarterly premiums on the index notional outstanding**
    - **Notional outstanding is reduced after each default**
      Notional reduction: weight of the defaulted name
      (e.g. iTraxx Europe: (1/125) x Index Notional)

- **Static credit portfolio**:
  existing contracts don’t roll automatically. **(What?)**

Index Mechanics

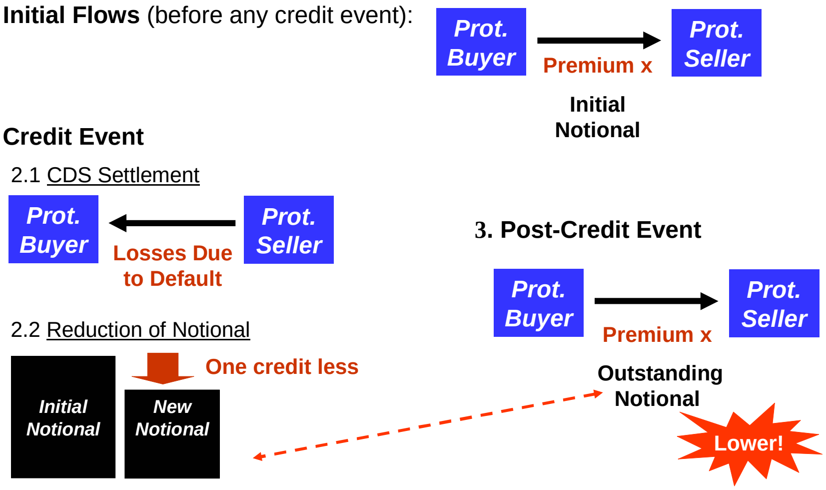
1. Initial Flows (before any credit event):

2. Credit Event

2.1 CDS Settlement

2.2 Reduction of Notional

3. Post-Credit Event



Static Portfolio

- **Static credit portfolio**:
  existing contracts don't roll automatically. **(What?)**

- iTraxx Europe started trading in July 2004.
- Each 20-Sept and 20-March there has been changes in the Index components
- CDS Indices do not work as equity indices in this respect

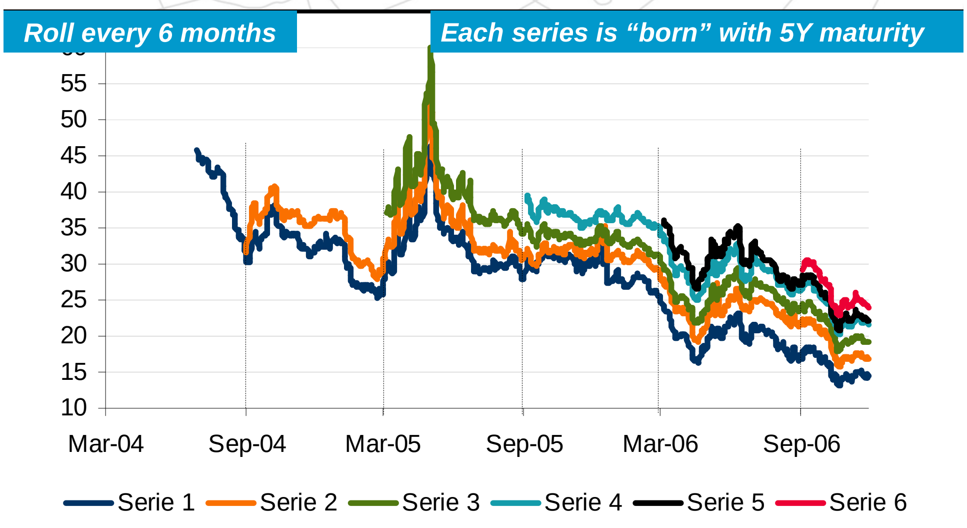
- What happens is that each 20-Sept and 20-March a new CDS Index **Series** is created
- This series co-exists with the previous ones
- At the moment we have 6 different iTraxx series!

On/Off The Run

Index	Tenor	Series	Start	Maturity
ITRAXX_EUROPE	5y	1	20-Mar-04	20-Jun-09
ITRAXX_EUROPE	5y	2	20-Sep-04	20-Dec-09
ITRAXX_EUROPE	5y	3	21-Mar-05	20-Jun-10
ITRAXX_EUROPE	5y	4	20-Sep-05	20-Dec-10
ITRAXX_EUROPE	5y	5	20-Mar-06	20-Jun-11
ITRAXX_EUROPE	5y	6	20-Sep-06	20-Dec-11

- A series goes "off-the-run" when it is no longer the latest one
- Investors typically "roll" to the "on-the-run" series
- Effectively, this means investing in a portfolio of credits with a constant 5y maturity

iTraxx Europe: 6 Series



Source: Bloomberg.

Global CDS Index Platform

Europe

- iTraxx Europe (125) — Inv. Grade
- iTraxx Crossover (45)
- SDI (75) — UK GBP
- iTraxx LevX Sub. (35) — Loan CDS Indices
- iTraxx LevX Senior (35) — Loan CDS Indices

North America

- CDX.NA.IG (125) — Inv. Grade.
- CDX.NA.HY (100) — High Yield
- CDX.NA.XO (35) — Crossover
- LCDX — Loan CDS Index (to be launched)

Asia

- iTraxx Japan (50)
- iTraxx ex-Japan (50)
- iTraxx Australia (25)

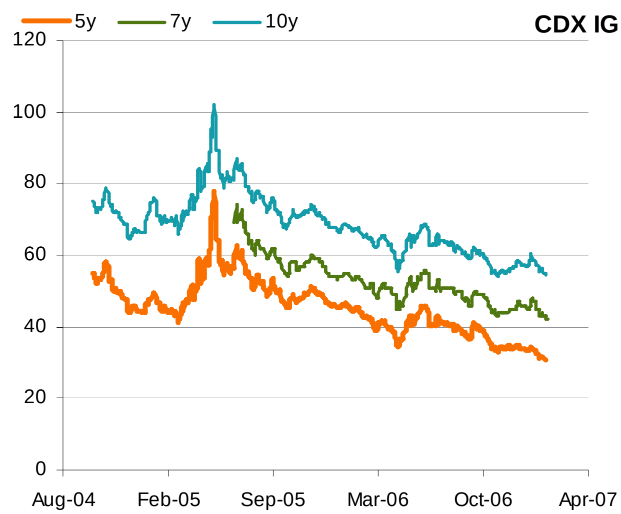
Emerging Markets

- CDX.EM (14)
- CDX.EM Diversified (40)

Indices (Pricing, Contracts, Members, Rules, Summary):
`http://www.markit.com/marketing/indices.php`

On The Run CDX.NA.IG: Historical Spreads

- 125 North American Credits
- Investment Grade
- Equally weigthed portfolio



Source: Bloomberg.

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CDOs & Tranches:

~25% of the Market

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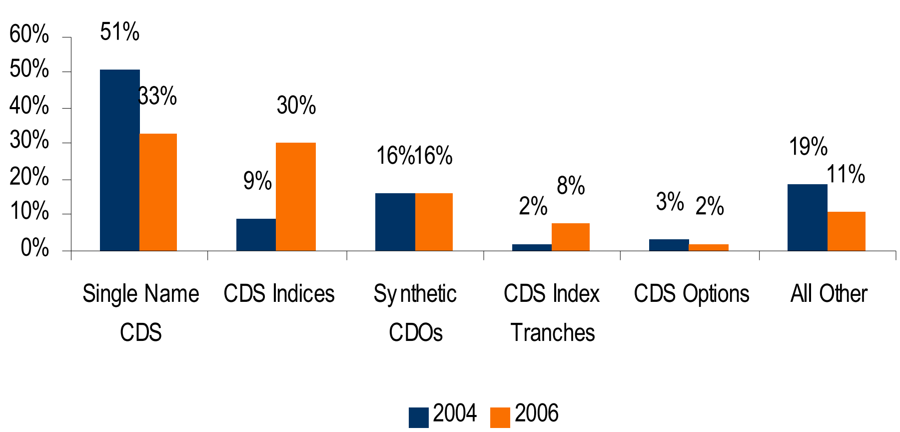
Credit Derivatives Market: Products

CDS & Indices: 66% Market – CDOs & Tranches: 24% Market



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Source: BBA 2006 Credit Derivatives Survey. 58

Is it that complex?

- Regulators and the media:
  - "Investors do not fully understand what they are purchasing in areas such as CDOs." (FT 2005)
  - "Understanding the credit risk profile of CDO tranches poses challenges even to the most sophisticated participants." (Alan Greenspan 2005)
  - "A number of investors and regulators have already voiced concern about the level of complexity in some investment products." (Nick Sawyer, Editor, Risk 2005)
- **It seems so**

Is it that complex?

- Risk Magazine (Feb'07):
  - The president of the ECB, Jean-Claude Trichet, has warned about the risks that certain exotic derivatives pose to the international financial system.
  - **"We are trying to understand what is going on – but it is a big, big challenge."**
  - Trichet's warnings echo a number of recent reports from regulators and central banks that have raised concerns about the stability of the credit derivatives market.
  - In December, the ECB voiced "unease" at the distribution of credit risk to "opaque" players such as hedge funds.

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- Lecture 1:
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Credit Risk Correlation

- **What is Credit Risk Correlation?**
  - Degree in which the credit risk of a group of firms is related
  - Different ways of measuring it

- **Is it important? Where does it play a role?**
  - Management of portfolios of defaultable instruments (bonds, loans, CDS, …)
  - Computing bank capital charges under Basel II
  - Pricing of multiname credit derivatives (CDOs, tranches, 1^{st} to default, …)

Credit Risk Correlation

- There are many papers dealing with credit risk correlations on different settings
- However, very few of them trying to estimate those correlations. Most of them are just "numerical" analysis about what happens if correlations take this or that value
- BIS (2003): "In practice, **default correlation has been difficult to estimate with any precision**."
- BIS (2005): "Although progress is being made, quantitative modeling of these correlations is complex and not yet fully developed."
- Schonbucher (2003): "**Default correlation and default dependency modeling is probably the most interesting and also the most demanding problem in the pricing of credit derivatives**."

Intensity Models: An example

- To introduce default correlation between two firms i and j we have to "correlate" their default intensities
- For example through a common factor

$$\lambda_{i,t} = a_i X_t + \varepsilon_{i,t}$$
$$\lambda_{j,t} = a_j X_t + \varepsilon_{j,t}$$

  - X_t is a factor affecting both intensities
  - $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ are independent and capture idiosyncratic credit risk.

Intensity Models: An example

$$\lambda_{i,t} = a_i X_t + \varepsilon_{i,t}$$
$$\lambda_{j,t} = a_j X_t + \varepsilon_{j,t}$$

- We can think of X_t as a factor representing the business cycle or state of the economy
- How important is X_t on the firm's credit risk?
- How much credit risk does it explain?
- Ten, forty, eighty percent?

Intensity Models: An example

- Elizalde (2005): "Do we need to worry about credit risk correlations?"
- Data:
  - 14 US firms: Owens, CSC, Comcast, Walt Disney, Ford Motor Credit, General Motors, Hertz, Marriott, Norfolk Southern, Sears Roebuck, TCI Communications, Time Warner, USX, Union Pacific
  - Daily bond prices from July'01 to November'03, the period with highest number of defaults and credit spreads for the last decade
- **A average fraction of the credit risk explained by a single common factor: 68%**

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Collateralized Debt Obligations

CDOs

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CDOs

- Imagine you have a portfolio of 125 bonds/loans of different European companies
- As an investor, your position is that of a “protection seller”, meaning that credit risk works against you
- Any default of a firm in your portfolio implies losses
- If you hold the same notional of each bond:
  - Your exposure to each firm is 0.8% of your total portfolio notional
  - If a firm defaults with a 40% recovery rate
  - How much do you lose?

    0.48% = Exposure (0.8%) * loss given default (60%)

CDOs

- You are exposed to the credit risk of all firms
- How can you hedge that?
  - Using CDS of each firm (buy protection)
  - Selling the portfolio in one piece
  - **Slicing (tranching) it and selling the pieces (tranches) to different investors**

- A CDO is a “device” to slice the credit risk of a portfolio into different “tranches” and to sell those tranches to different investors
- That definition doesn’t help very much, Does it?

CDO: An Example

- Three groups of investors:
  - "**Equity**" investors:
    - They will pay me all the losses in the portfolio up to 5% (around 10 defaults)
    - Once the losses exceed 5%, they stop paying.
  - "**Mezzanine**" investors:
    - They will not pay anything until losses reach 5% of the portfolio
    - They will pay all losses exceeding 5% up to 15%
    - Once the losses exceed 15%, they stop paying.
  - "**Senior**" investors: pay anything above 15%.

CDO: An Example

- Portfolio of 125 bonds/loans
- Each default implies a loss of 0.48% of the total notional
- Imagine the portfolio generates 2% a year in coupons

- I have passed all losses to investors by slicing them and selling them in "tranches":
  - Equity (0-5%)
  - Mezzanine (5-15%)
  - Senior (15%-100%)
- I have no further exposure to defaults.
- **Deal?**

CDO: An Example

- **What do we do with the coupons the bonds generate?**
- We also distribute them among tranche investors
- How? In a way that compensates them for the risk they are taking

- **The price** (also called premium or spread) **of each tranche refers to the amount of money tranche protection sellers receive for the risk the are taking**

- Such a spread is expressed in an annual % of the tranche notional and is paid quarterly

CDO: An Example

- **Which tranche is riskier?**
- **Which are the risk factors?**
  - Default probabilities
  - Recovery rates
  - Default correlations

- Let's say you sold protection on the equity tranche. Would you require more or less spread if default probabilities go up? If recovery rates go down? If default correlations go up?
- And if you sold protection on the Senior tranche?

CDO: An Example

- How do you construct a model to price a tranche?
- What should you take into account?

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CDO Pricing

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CDOs

- In a CDO, a pool of defaultable instruments (Bonds, Loans, CDS, LCDS, …) is put together and sold in tranches.
- Each tranche is characterized by an attachment and detachment points specifying the range of portfolio losses it will absorb, i.e. its subordination in the capital structure.

Tranche	K_L Attachment	K_U Detachment
Equity	0%	5%
Mezzanine	5%	15%
Senior	15%	100%

CDOs

- Some notation:
  - t • Time, in years, since the beginning of the contract
  - T • Maturity, in years, of the CDO
  - M • Initial notional of the portfolio
  - η • Frequency of premium payments (i.e. to the tranche investor or protection seller)
  - Z_t • **Percentage loss in the portfolio** value at time t

CDOs

- Terminology
  - Sell tranche protection = Long tranche
    - Receive premiums, pay losses
  - Buy tranche protection = Short tranche
    - Pay premiums, receive losses

- **Losses suffered by holder of tranche *j*** since origination until time *t* :

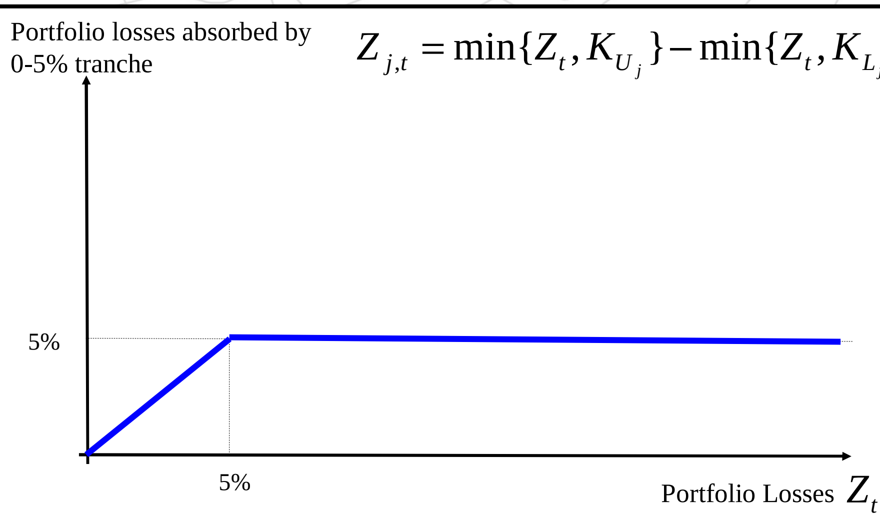
$$Z_{j,t} = \min\{Z_t, K_{U_j}\} - \min\{Z_t, K_{L_j}\}$$

(expressed as % of original portfolio notional M)

CDOs

0-5% Equity Tranche Loss Profile

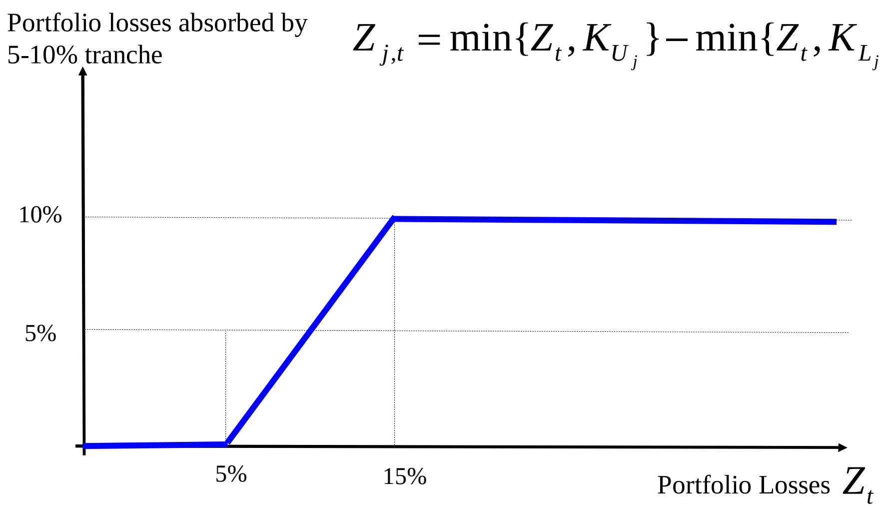
$$Z_{j,t} = \min\{Z_t, K_{U_j}\} - \min\{Z_t, K_{L_j}\}$$



CDOs

5-15% Mezz. Tranche Loss Profile

$$Z_{j,t} = \min\{Z_t, K_{U_j}\} - \min\{Z_t, K_{L_j}\}$$

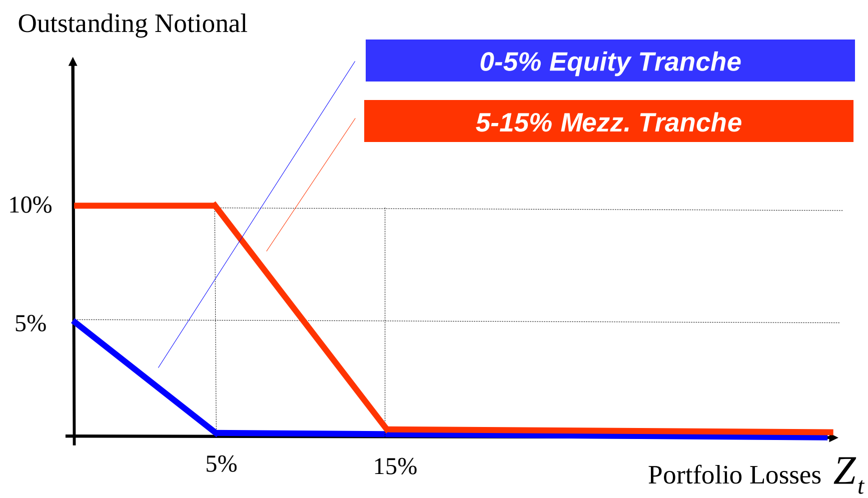


CDOs

- The tranche protection seller
  - Pays the tranche losses (due to defaults) at the time of defaults
  - Receives premium payments with a quarterly frequency $(\eta = 0.25)$
- The premium s_j is the "price" of the tranche an is **fixed at the beginning** of the contract
- **The premium payments are equal to the premium times the tranche outstanding notional**

$$\Gamma_{j,t} = (K_{U_j} - K_{L_j} - Z_{j,t})M$$

CDOs



CDOs

- Tranche Outstanding Notional

$$\Gamma_{j,t} = (K_{U_j} - K_{L_j} - Z_{j,t})M =$$

$$= \begin{cases} (K_{U_j} - K_{L_j})M & \text{if } Z_t < K_{L_j} \\ (K_{U_j} - Z_{j,t})M & \text{if } K_{L_j} < Z_t < K_{U_j} \\ 0 & \text{if } Z_t > K_{U_j} \end{cases}$$

(Equity) Tranche Mechanics

1. Initial Flows (before any credit event):

Prot. Buyer → Premium x Initial Notional → Prot. Seller

2. Credit Event

2.1 CDS Settlement

Prot. Buyer ← Losses Due to Default ← Prot. Seller

2.2 Reduction of Notional

Initial Notional — Losses Due to Default → New Notional

3. Post-Credit Event

Prot. Buyer → Premium x Outstanding Notional → Prot. Seller

Lower!

CDOs

- Assume defaults only happen at premium payment dates
- At each premium payment date $t = \eta, 2\eta, \ldots, T$

  Tranche investors:

  – Pay any losses realized during the period due to defaults

$$(Z_{j,t} - Z_{j,t-\eta})M$$

  – Receive a premium s_j of the outstanding notional

$$s_j \eta \Gamma_{j,t}$$

Pricing CDOs

- Find the "fair" premium s_j for each tranche j
- "Fair" means that the net (expected) present value of the cash flows paid and received by the tranche is zero
- A CDO has two legs or cash flow streams:
  - **Premium Leg**: premiums received by protection seller
  - **Default Leg**: losses paid by protection seller
- The premium is fixed so the expected value of both legs is the same

Pricing CDOs

- Consider a CDO with:
  - Payment dates $t_1, t_2, ..., t_N$
  - Maturity t_N
  - Notional M

  where $\eta = t_{n+1} - t_1$ for all $n = 0, 1, ..., N$
- The contract starts at time $t_0 = 0$
- The first premium is due at t_1

Premium Leg

- Expected payments to the tranche holder:
  - At t_1 : $s_j \eta \, E[\Gamma_{j,t_1}]$
  - …
  - At t_N: $s_j \eta \, E[\Gamma_{j,t_N}]$

- Present value of the premium leg of tranche *j*:

$$PL_j = \sum_{n=1}^{N} DF(t_0, t_n) \cdot s_j \cdot \eta \cdot E[\Gamma_{j,t_n}]$$

$$\Gamma_{j,t} = (K_{U_j} - K_{L_j} - Z_{j,t})M$$

Default Leg

- Expected payments by the tranche holder:
  - At t_1 : $E[(Z_{j,t_1} - Z_{j,t_0})M]$
  - …
  - At t_N: $E[(Z_{j,t_N} - Z_{j,t_{N-1}})M]$

- Present value of the premium leg of tranche *j*:

$$DL_j = \sum_{n=1}^{N} DF(t_0, t_n) \cdot E[(Z_{j,t_n} - Z_{j,t_{n-1}})M]$$

Pricing CDOs

- Making **PL=DL** and rearranging

$$s_j = \frac{\sum_{n=1}^{N} DF(t_0,t_n)\cdot(E[Z_{j,t_n}]-E[Z_{j,t_{n-1}}])}{\sum_{n=1}^{N} DF(t_0,t_n)\cdot\eta\cdot(K_{U_j}-K_{L_j}-E[Z_{j,t_n}])}$$

- Where

$$E[Z_{j,t_n}] = E[\min\{Z_{t_n},K_{U_j}\}-\min\{Z_{t_n},K_{L_j}\}]$$

- **Need to know the distribution function of the total portfolio losses Z_T at each time $T=t_1,t_2,...,t_N$**

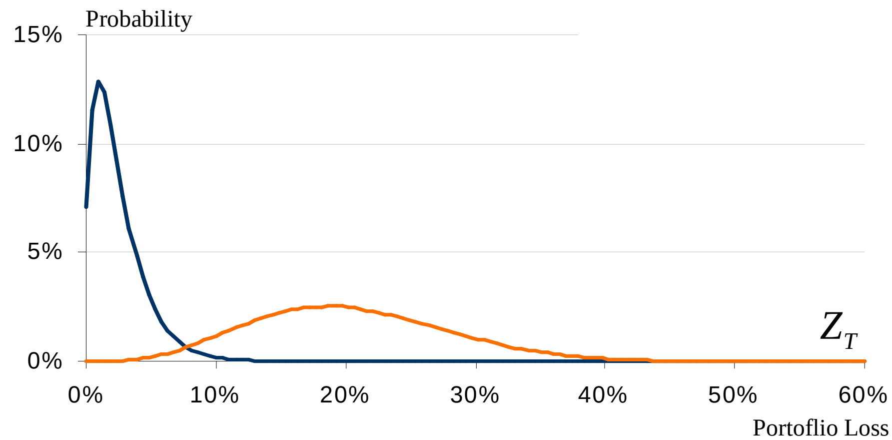
Remember iTraxx Europe?

- Portfolio of 125 European investment grade credits
- Sector and country diversified
- Imagine the portfolio behind our CDO is iTraxx Europe
- To price tranches we **need to know the distribution function of the total portfolio losses of iTraxx Europe at each time** (until maturity):
  - For each future time (3 months, 6 months, …, 5 years) we are looking for
  - Probability of each level of portfolio losses, i.e.
  - Probability of having 1, 2, …, 125 defaults

- **How would go about it?**

Portfolio Loss Distribution Function

Two examples of how it can look like



Distribution Function of Portfolio Losses

- Therefore we just **need to know the distribution function of the total portfolio losses at each future time**
- There are many ways to derive that distribution, which is the key modeling challenge in CDO pricing
- We will review the simplest version of the most widely used way of deriving the distribution function of the total portfolio losses:

Asymptotic Single Risk Factor Model

Distribution Function of Portfolio Losses:

From Structural Models to Gaussian Copula

Merton Model: Firm *i*

- Take the firm today at time $t = 0$

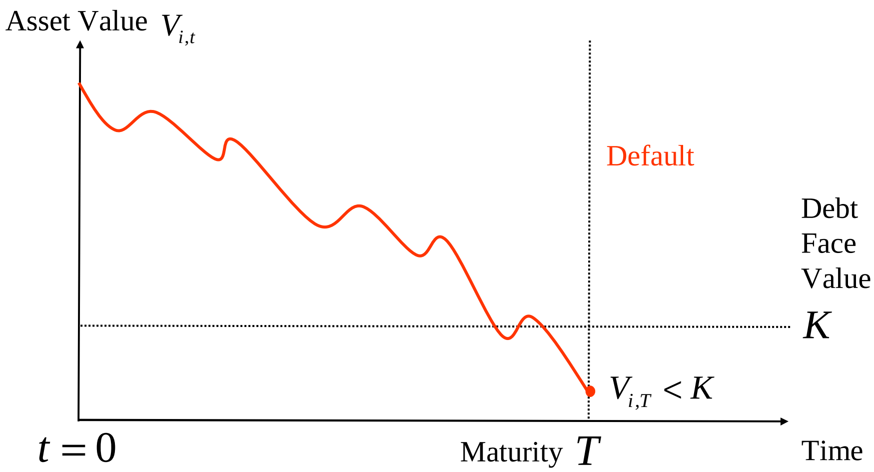
Assets	Debt
	Equity

- Asset value follows a GBM

$$dV_{i,t} = rV_{i,t}dt + \sigma_i V_{i,t} dW_{i,t}$$

- Assume debt is composed of a zero-coupon bond with
  - Maturity T
  - Face Value K_i

Merton Model: Firm *i*



Merton Model: Firm *i*

- Probability of default before time T

$$P[V_{i,T} < K_i]$$

- Using Ito's lemma

$$V_{i,T} = V_{i,0} e^{(r - \sigma_i^2/2)T + \sigma_i \sqrt{T} X_{i,T}}$$

where

$$X_{i,T} = \frac{W_{i,T} - W_{i,0}}{\sqrt{T}}$$

follows a N(0,1).

Merton Model: Firm *i*

- Default before time T i.f.f. $V_{i,T} < K_i$ or equivalently i.f.f.

$$X_{i,T} < \frac{\ln K_i - \ln V_{i,0} - (r - \sigma_i^2/2)T}{\sigma_i \sqrt{T}}$$

- Default probability

$$p_{i,T} = P[V_{i,T} < K_i] = \Phi\left(\frac{\ln K_i - \ln V_{i,0} - (r - \sigma_i^2/2)T}{\sigma_i \sqrt{T}} \right)$$

Φ : cdf of a N(0,1)

Merton Model: Firm *i*

- Merton (74) model gives us the default probability for each firm *i*.
- But … we are looking for the distribution function F of the portfolio loss rate Z_t which depends on
  - Default probability of each firm
  - Recovery rate of each bond
  - Default correlation between the firms
- We follow Vasicek (1987, 1991, 2002) to derive the portfolio loss distribution F

Vasicek: Portfolio Loss Distribution

- Portfolio of I bonds of different firms
- **How do we introduce correlation between the credit risk of the firms?**
- Remember, firm i defaults at time T if

$$\underbrace{X_{i,T}}_{N(0,1)} < \underbrace{\frac{\ln K_i - \ln V_{i,0} - (r - \sigma_i^2/2)T}{\sigma_i \sqrt{T}}}_{\text{Constant}}$$

Vasicek: Portfolio Loss Distribution

- Assume the random variables $X_{1,T},\dots,X_{i,T},\dots,X_{I,T}$ are correlated
- Assume they are function of two factors:
  - A common factor Y_T
  - An idiosyncratic factor $\varepsilon_{i,T}$

all i.i.d. N(0,1) distributed

$$X_{i,T} = \sqrt{\rho} \cdot Y_T + \sqrt{1-\rho} \cdot \varepsilon_{i,T}$$

"**Single Risk Factor**" model to introduce correlation

ρ : **exposure to common factor, same for all firms**

Vasicek: Portfolio Loss Distribution

- "**Single Risk Factor**" model to introduce correlation

$$X_{i,T} = \sqrt{\rho} \cdot Y_T + \sqrt{1-\rho} \cdot \varepsilon_{i,T}$$

  - ρ controls the correlation between the firms' default probabilities; It can be thought as the exposure to the state of the economy (approximated by Y_T)
  - But … if you think it twice, it's a little bit … well … not very clear what it is.

$X_{1,T}, \ldots, X_{i,T}, \ldots, X_{I,T}$ follow a multinomial normal random variable, i.e. a **Gaussian copula**

Vasicek: Portfolio Loss Distribution

- Summing up; so far we know
  - The firms' default probabilities $p_{1,T}, \ldots, p_{i,T}, \ldots, p_{I,T}$
  - How they are correlated (single risk factor) ρ
- We were looking for the distribution function F of portfolio losses
- Three more assumptions and we are done.
  (Without going into the maths).

Vasicek: Portfolio Loss Distribution

- Assumptions
  - The default probability of all firms is the same p_T
  - The number of bonds in the portfolio is very large $I \to \infty$
  - The recovery rate of all bonds is R
- That, and a little bit of maths, gives us the distribution function of portfolio losses

(see survey IV @ abelelizalde.com)

Vasicek: Portfolio Loss Distribution

- Distribution function of portfolio loss rate Z_T at time T :

$$F(Z_T \mid p_T, R, \rho) = (1-R) \cdot \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(Z_T) - \Phi^{-1}(p_T)}{\sqrt{\rho}} \right)$$

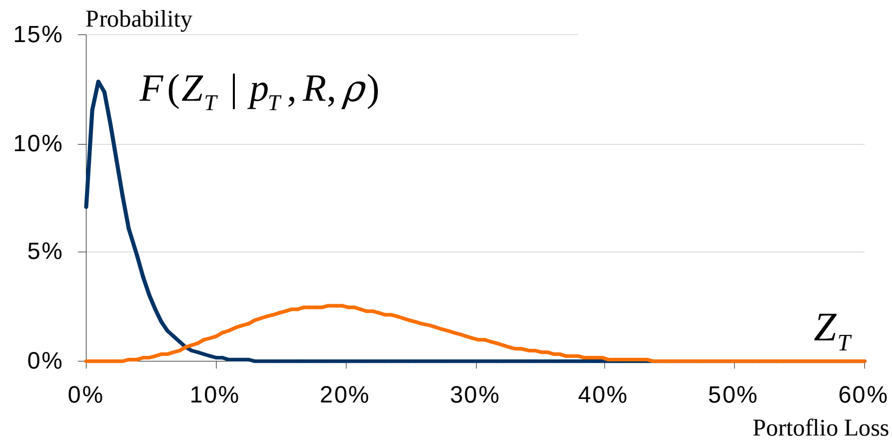
using the "Asymptotic Single Risk Factor" model

- Since the dependence is introduced through normal random risk factors, this family of models is referred to as "**Single Factor Gaussian Copula Model**"

Φ : cdf of a N(0,1)

Portfolio Loss Distribution Function

That's it. ***How does F depend on each risk factor?***



Pricing CDOs

- Remember

$$s_j = \frac{\sum_{n=1}^{N} DF(t_0,t_n)\cdot(E[Z_{j,t_n}]-E[Z_{j,t_{n-1}}])}{\sum_{n=1}^{N} DF(t_0,t_n)\cdot\eta\cdot(K_{U_j}-K_{L_j}-E[Z_{j,t_n}])}$$

where

$$E[Z_{j,t_m}] = E[\min\{Z_{t_n}, K_{U_j}\} - \min\{Z_{t_n}, K_{L_j}\}]$$

for $t_1,\ldots,t_n,\ldots,t_N$

Pricing CDOs

- We just have to evaluate

$$E[Z_{j,t_m}] = E[\min\{Z_{t_n}, K_{U_j}\} - \min\{Z_{t_n}, K_{L_j}\}]$$
$$= \int_0^1 (\min\{z, K_{U_j}\} - \min\{z, K_{L_j}\}) dF(z \mid p_t, R, \rho)$$

for all $t_1, \ldots, t_n, \ldots, t_N$ using a numerical procedure

- And we are done, we know the fair premium of our tranches
- It only requires to (numerically) solve some integrals

ASRF Model: Assumptions

1. Merton and Vasicek models are good representions of reality (?)
2. All random variables considered (e.g. common factor) are Gaussian
3. Exposure of all firms to the common factor is the same (?)
4. Recovery rate is the same for all bonds (?)
5. Default probability of all firms is the same (?)
6. Number of bonds in our portfolio is very large (?)
7. All bonds have the same weight in the portfolio (?)

Assumptions 4-7 are easily relaxed without too much mathematical complexity, which is the standard practice.

"Standard" model

- The previous model, assuming:
  - Default probability of all firms is not necessarily the same
  - Number of bonds in the portfolio is finite
  - The weight of each bond in the portfolio is not necessarily the same
  - Recovery rates can be different across bonds (though independent of default probabilities)

  is the standard model in the industry, with "regional" variations and refinements.
- It can be easily coded in VBA, Matlab, C++, …
- The industry is continuously working on a definite and better model to substitute it. Many improvements and advances have been made.

Extensions

- The previous model has been extended in several different directions to make it more realistic.

  Among them:
  - Other copulas, i.e. distribution functions for the common factor: T-student, … all you can think of
  - Correlations varying with time, with the common factor, random, …
  - Several, rather than just one, common factors
  - …

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Cash vs. Synthetic CDO

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Cash vs Synthetic CDO

- **Cash CDO**: So far, we have “tranched” a portfolio of bonds (cash or “real” instruments)

- One can do exactly the same with a portfolio of CDS
  - **CDS give you the same credit exposure than bonds**, so it can be tranched in the same way
  - CDS generate a premium which can be distributed among tranche investors in the same way

  This way, we create a **Synthetic CDO**

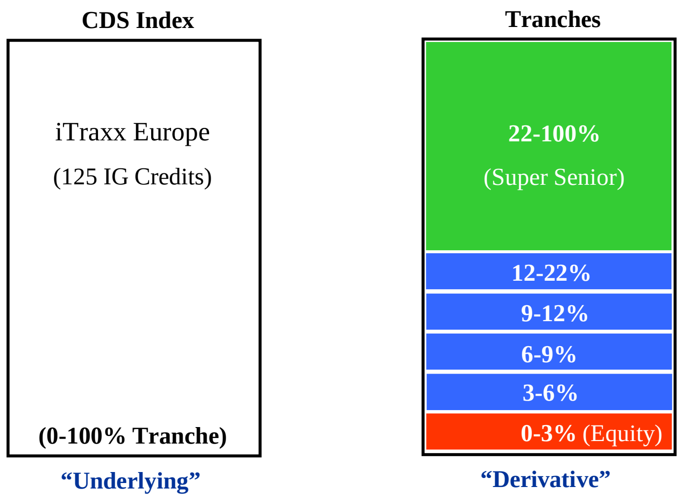
CDS Index

- **A CDS Index is simply a portfolio of CDS, untranched**
- It trades in the market in the same way as the S&P500 does (an share, rather than CDS, index)
- One can buy or sell protection on a CDS index
- An index position has, again, two legs (like a CDS or a CDO):
  - Default Leg: each default generates a loss to the protection seller
  - Premium Leg: the index trades with a coupon which is paid to the protection seller
- A CDS Index is like a single name CDS, but referring to a portfolio of credits

CDS Index

- Synthetic CDO: tranche a portfolio of CDS
- CDS Index: portfolio of CDS
- So we can use a CDS Index (iTraxx, CDX IG, …) to create tranches referring to it
  - The market agrees on a set of “standard” attachment and detachment points
  - iTraxx tranches: 0-3%, 3-6%, 6-9%, 9-12%, 12-22%
  - Everybody trades these tranches, which increases the liquidity of the market and the ability of unwinding positions
  - Bespoke (e.g. 4-7%) can be created, but they are not liquid

Standard Tranches iTraxx Europe



A reduced history of tranches

- Banks start creating Cash CDOs to reduce their credit exposure
- Investment banks create cash CDOs to sell among clients
  - To create a cash CDO you have to put together a portfolio of bonds, which can take time
  - Doing it with CDS takes nothing, so synthetic CDOs start becoming popular
- Investment banks need to hedge the CDO tranches they sell: they can hedge the "default" risk with CDS or indices but, How do they hedge the correlation exposure?
  - Standard tranches on CDS indices are born,
  - i.e. synthetic tranches with an underlying CDS portfolio

Correlation, Recovery and Default Probabilities

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Portfolio Loss Distribution

- Pricing tranches only requires us to compute the distribution function $F(\cdot)$ of the portfolio losses Z_t at each time $t = t_1, \ldots, t_N$
- Independently of the model/assumptions we use, the portfolio loss distribution will depend on:
  - Credits' default probability
  - Credits' recovery rate
  - Credits' default correlation
- How does that dependence look like?

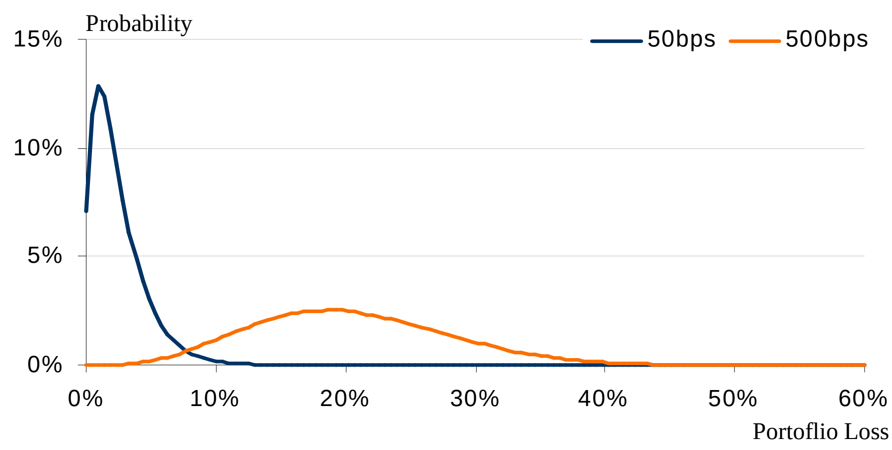
Portfolio Loss Distribution

- Let us use the following model:
  - Single risk factor model (not asymptotic)
  - Underlying portfolio of 125 CDS
  - All credits have the same:
    - Weight in the portfolio (0.8%)
    - Recovery rate (40%)
    - CDS spread (50 bps), i.e. default probability
    - Default correlation ρ (10%)

- We show next how the portfolio loss distribution varies when we change CDS spreads, recovery rates and default correlation

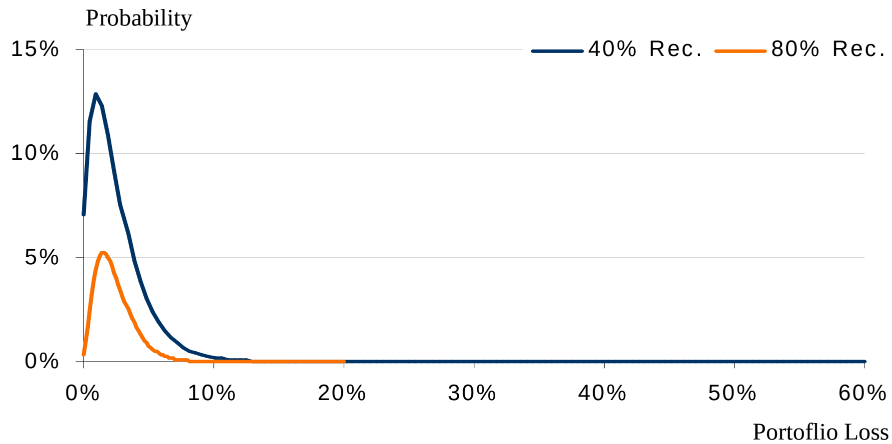
Portfolio Loss & Default Probability

Higher default probability increases the probability of higher losses



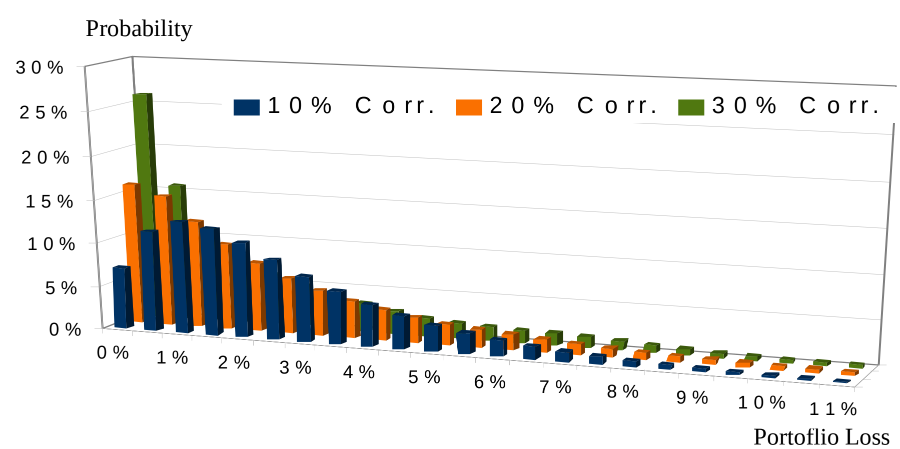
Portfolio Loss & Recovery Rate

Higher recovery decreases the probability of higher losses



Portfolio Loss & Recovery Rate

Correlation increases the probability of extremelly low and high losses

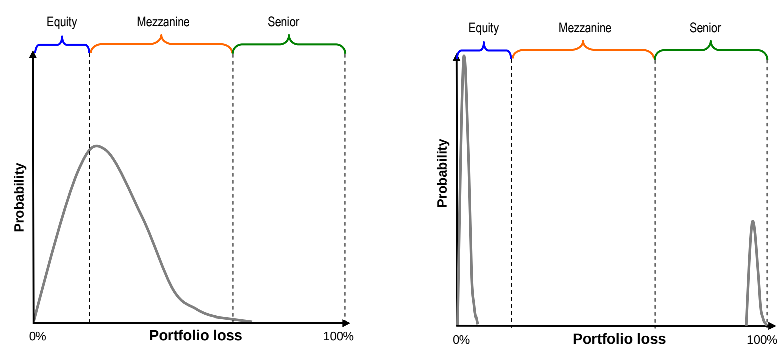


Tranches

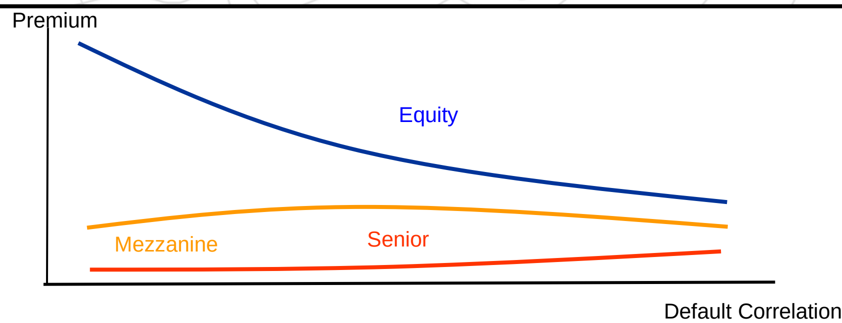
- Sensitivities: (EL: expected losses)
  - Higher default prob. ⇨ Higher EL for all tranches
  - Lower recovery rates ⇨ Higher EL for all tranches
  - The impact of correlation varies across tranches:
    A higher correlation decreases the EL of the equity tranche but increases the EL of the senior tranches
- The higher the EL of a tranche, the higher its premium

Standard Tranches

- Correlation impacts how losses are distributed throughout the portfolio
  - Low correlation (left): loss distribution smooth across junior and mezzanine tranches
  - High correlation (right): losses pushed towards senior tranches



Tranche premium vs. default correlation



Default correlation will determine the shape of the loss distribution, which is the key determinant of how much losses each tranche absorbs.

Default correlation does not affect the total expected losses, and as a consequence it does not affect the index itself.

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Model Calibration

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iTraxx Europe 5Y Series 6 Quotes

Index (0-100%)**:** 23 bp

Tranches:

0% - 3%	500 bp + 10.25% Upfront
3% - 6%	46 bp
6% - 9%	13.5 bp
9% - 12%	4.5 bp
12% - 22%	2.25 bp

Super Senior 22-100%: not usually quoted, difference between the index and the rest of the tranches

Data as of 5-Jan-2007

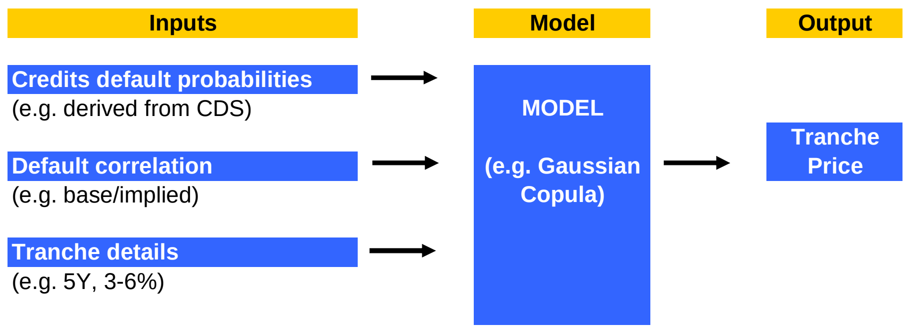
How many defaults can we take?

Tranche	Defaults to be hit	Defaults to wipe out
0-3%	1	7
3-6%	7	13
6-9%	13	19
9-12%	19	25
12-22%	25	46

iTraxx Europe tranches

(assuming a 40% recovery rate)

Valuation of Tranches



What do we need to price a tranche?

- Tranche characteristics: Maturity (e.g. 5y) and subordination (e.g. 3-6%)
- **Recovery rate**: deterministic assumption (40%) market standard
- **Default probabilities**:
  - Can be obtained using a reduced or structural model
  - Usual practice: calibrate an intensity model to the firm's CDS spreads
- **Default Correlation**:
  - Well …

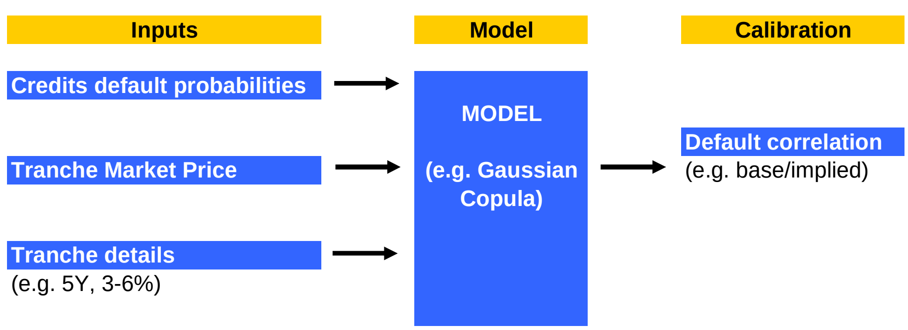
Default Correlation?

- In the single risk factor copula model, default correlation is introduced through the parameter ρ

  … which represents … (?)

- It is not very clear what it exactly represents, though it determines the firms' default correlation
- As such, it is not very practical to estimate it
- But we can calibrate it, can't we?

Default Correlation Calibration



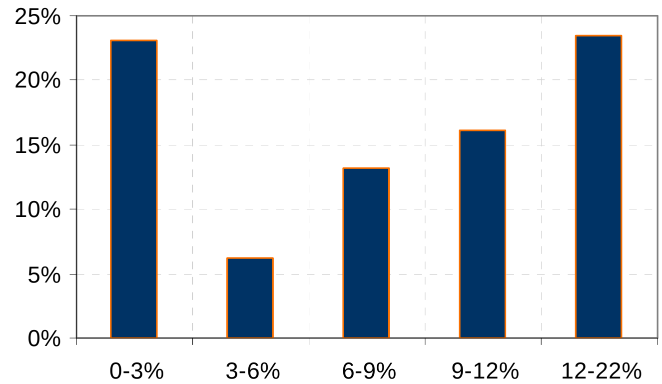
Implied Correlation

Implied Correlation

- For each tranche we know:
  - Default probabilities
  - Recovery rates
  - Model we use
  - Market price
- The only free parameter is the correlation parameter, which we can calibrate such that the model gives us the exact market price
- So we have an implied correlation for each tranche
- If the model is correct, all implied correlations should be the same. However …

Implied Correlation iTraxx 5Y

Implied Correlation	
0-3%	**23.1%**
3-6%	**6.2%**
6-9%	**13.2%**
9-12%	**16.1%**
12-22%	**23.5%**



Why?

- Obviously, the model is not perfect
- How can we explain the “correlation smile” or “skew”?
- Amato and Gyntelberg (2005) provide some insights:
  - Segmentation among investors across tranches, and these investor groups hold different views about correlations
  - The smile reflects market participants’ uncertainty about how best to model credit risk correlations

    (e.g. equity tranches contain a “model risk premium”)
  - Local demand conditions in prices
  - The market participants might be using other models

Problems of Implied Correlation

- Many, among others:
  - We can only guarantee its existence for the equity tranche
  - We can only guarantee its uniqueness for the equity tranche

- This poses important challenges to day-to-day market practices:
  - Compute the sensitivity of mezzanine tranches to correlation
  - Prices bespoke tranches by interpolating correlations on standard tranches

Problems of Implied Correlation

- We have a position on a mezzanine tranche and we want to know how sensitive it is to default correlation
  - Do we make or lose money if correlations increase?
  - For that, we find out the implied correlation
  - And we shock it: we compute the price of the tranche by assuming a slightly higher correlation keeping all the other parameters (probabilities, recoveries, …) constant

- We compute its implied correlation and … it has two implied correlations! … or … it has none!

- Which one do we shock?

Base Correlations

Base Correlation

- Base correlations are "cousins" of implied correlations which solve some of their problems (but introduce others)
- They have replaced implied correlations as the market standard to talk about default correlation
- Idea:
  - Implied correlations do not give problems for equity tranches: they exist and their impact is monotonic
  - Any tranche (e.g. long 3-6%) can be expressed as two equity tranches (long 0-6% & short 0-3%)

Base Correlation

- Idea:
  - Implied correlations do not give problems for equity tranches: they exist and their impact is monotonic
  - Any tranche (e.g. long 3-6%) can be expressed as two equity tranches (long 0-6% & short 0-3%)
- Base correlation is the correlation of base tranches: 0-3%, 0-6%, 0-9%, …
- Implied correlations referred to 0-3%, 3-6%, 6-9%, …

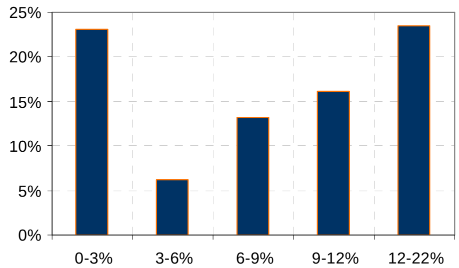
Base Correlation

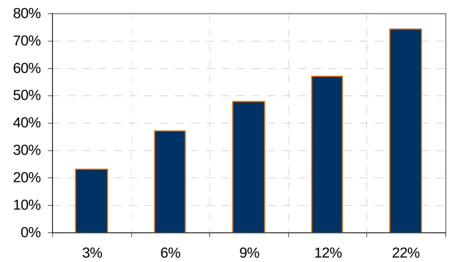
- One can use the same models
- Base correlations are by definition correlations of equity tranches: unique and monotonic impact
- They solve some of the problems of implied correlations
- But not all, and are even more **difficult to interpret** or understand

Implied vs Base Correlation iTraxx

Implied Correlation	
0-3%	**23.1%**
3-6%	**6.2%**
6-9%	**13.2%**
9-12%	**16.1%**
12-22%	**23.5%**

Base Correlation	
3%	**23.1%**
6%	**37.2%**
9%	**48.2%**
12%	**57.3%**
22%	**74.5%**





Current Modelling Efforts

Bottom-Up Models

- Remember that we only **need the distribution function of portfolio losses**
- Bottom-up models construct it starting from the default probabilities, recovery rates and correlations of the firms in the portfolio (e.g. single factor copula model)
- Single factor models are not completely satisfactory in several dimensions, which drive current research:
  - Base/Implied correlations can be hard to calibrate/interpret
  - Most models are static and therefore not suitable to price products such as options on tranches, forward tranches, … whose payoffs depend on the dynamics of tranche spreads

Top-Down Models

- Remember that we only **need the distribution function of portfolio losses**
- Why bother with the underlying credits? Let's model the portfolio loss directly
- Advantages:
  - Dynamic models
- Disadvantages:
  - De-linked from the sources of credit risk: firms
  - Not generally suited for bespoke pricing

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What about May'05?

King's College London - 2007

www.abelelizalde.com

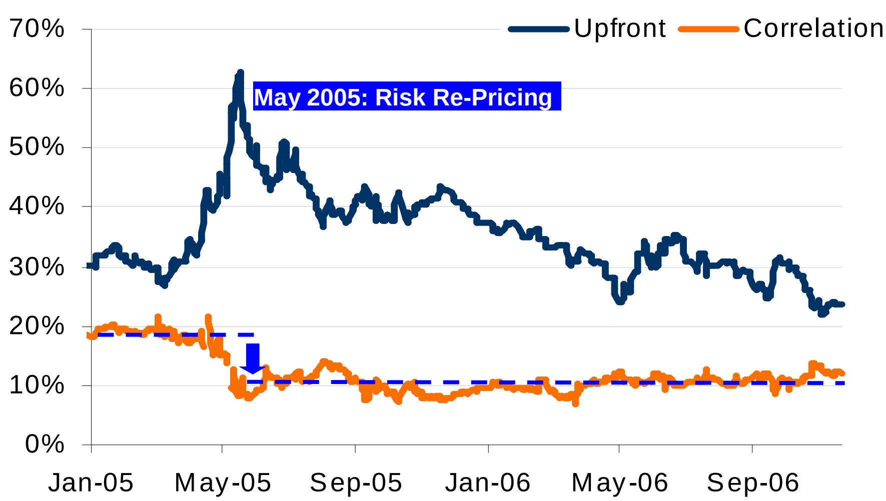
May 2005: Early Days of Correlation

- The standardized tranche market develops due to dealers' hedging needs
  - Dealers had sold bespoke tranches to real money investors (mostly senior mezzanine)
  - Their position was long spreads and long correlation
  - Spread exposure was hedged with indices
  - Correlation exposure? Standardized tranche market grows
- Dealers needed to enter short correlation trades
  (e.g. buy equity protection)
- Hedge funds willing to take the other side

May 2005: Correlation Shakeout

- Before May 2005
  - Long correlation trades became attractive
  - Sell equity protection (delta hedged with mezzanine)
  - Positive carry (or generous upfront)
  - Positive MTM on correlated spread changes
  - Danger: idiosyncratic credit events
- Ford and GM downgraded on May 2005
  - Market realizes that a default can happen at any time
  - Long correlation trades suffer
  - Hedge funds try to unwind their positions
  - Unhedged dealers try to hedge
- Equity spreads widen dramatically

CDX IG 5Y Equity Tranche Spreads



iTraxx 5Y Equity Tranche Spreads